[21] P. Z. Revesz. On the Semantics of Theory Change: Arbitration between Old and New Information. In *ACM Symposium on Principles of Database Systems*, pages 71–82, 1993.

[22] J. Ullman. *Principles of Database and Knowledge-Base Systems, Vol. I.* Computer Science Press, 1988.

Appendix: Proofs of Results

Proof of Theorem 1: We show that a formula ϕ is not finitely satisfiable iff there is no minimal repair for ϕ viewed as an integrity constraint and for a given database instance. The latter problem is known not to be r.e.

Assuming an arbitrary (finite) database instance r, notice that ϕ is not finitely satisfiable iff there is no repair r' of r. Then there is no minimal repair of r. Conversely, assume there is no minimal repair of r. Then assuming there is a (non-minimal) repair r' of r, there is a minimal instance r'', $r'' <_r r'$, which is a repair of r. This follows from the fact that both r and r' are finite. Thus a contradiction is obtained.

Lemma 1. If $r \models \mathrm{T}_\omega(l(\bar{a}))$, where $l(\bar{a})$ is a ground literal, then for every repair r' of r, it holds $r' \models l(\bar{a})$.

Proof: By contradiction, assume that there is a repair r' of r such that $r' \not\models l(\bar{a})$.

Define a sequence of sets of ground literals:

$$\begin{aligned}\Sigma_0 :=\ & \{l(\bar{a})\},\\ \Sigma_{n+1} :=\ & \{l'(\bar{b}) | r \models \mathrm{T}_\omega(l'(\bar{b})) \text{ and there exists a literal } l^*(\bar{c}) \in \Sigma_n, \text{ such that } l'(\bar{b}) \text{ appears in one of the residues of } l^*(\bar{c}) \text{ when they are instantiated on the domain of the database}\},\end{aligned}$$

and finally $\Sigma_\omega := \bigcup_{n<\omega} \Sigma_n$.

Now define a new structure r^* as follows: For a base predicate p, if $p(\bar{c}) \in \Sigma_\omega$ or $\neg p(\bar{c}) \in \Sigma_\omega$, then $r^* \models p(\bar{c})$ iff $r \models p(\bar{c})$. If $p(\bar{c}) \notin \Sigma_\omega$ and $\neg p(\bar{c}) \notin \Sigma_\omega$, then $r^* \models p(\bar{c})$ iff $r' \models p(\bar{c})$.

This structure has the following properties:

(I) r^* is a database instance. For this we only need to prove that r^* satisfies only a finite number of positive ground literals. By its definition, r^* can be obtained adding to r' the positives literals in Σ_ω and removing from r' the negatives literals in Σ_ω. But $r \models \Sigma_\omega$, thus in Σ_ω there is a finite number of positive literals. Therefore, we added to r' a finite number of positives literals to obtain r^*, thus r^* is a database instance.

(II) $\Delta(r, r^*) \subsetneq \Delta(r, r')$

1. First, we prove that $\Delta(r, r^*) \subseteq \Delta(r, r')$. Let $p(\bar{a}) \in \Delta(r, r^*)$. We have to consider two cases.
   (a) If $r \models p(\bar{a})$ and $r^* \models \neg p(\bar{a})$, then $p(\bar{a}) \notin \Sigma_\omega$ (since if $p(\bar{a}) \in \Sigma_\omega$, then $r^* \models p(\bar{a})$) and $\neg p(\bar{a}) \notin \Sigma_\omega$ (since $r \models \Sigma_\omega$). Therefore, $r^* \models p(\bar{a})$ if and only if $r' \models p(\bar{a})$. Thus $r' \models \neg p(\bar{a})$ and $p(\bar{a}) \in \Delta(r, r')$.
   (b) If $r \models \neg p(\bar{a})$ and $r^* \models p(\bar{a})$, then as in the previous case, we conclude that $p(\bar{a}) \in \Delta(r, r')$.
2. If $l(\bar{a})$ is $p(\bar{a})$ or $l(\bar{a})$ is $\neg p(\bar{a})$, we have that $p(\bar{a}) \notin \Delta(r, r^*)$ and $p(\bar{a}) \in \Delta(r, r')$, therefore using 1 we conclude that $\Delta(r, r^*) \subsetneq \Delta(r, r')$.

(III) $r^* \models IC$. Let IC_D be the set constructed as follows: for every integrity constraint $\forall(\bigvee_{i=1}^m l_i(\bar{x}_i) \vee \psi(\bar{x}))$ in IC, instantiate it on the domain of a database obtaining formulas of the form $\bigvee_{i=1}^m l_i(\bar{a}_i) \vee \psi(\bar{a})$. If $r \models \neg\psi(\bar{a})$, then include $\bigvee_{i=1}^m l_i(\bar{a}_i)$ in IC_D.

(III.I) If $l'(\bar{b}) \in \Sigma_\omega$, then for every element in IC_D of the form $\neg l'(\bar{b}) \vee \bigvee_{i=1}^m l_i(\bar{b}, \bar{a}_i)$, it holds $m \geq 1$ and $l_j(\bar{a}_j) \in \Sigma_\omega$, for some $1 \leq j \leq m$. This is saying that m cannot be 0, in other words, $\neg l'(\bar{b})$ cannot appear isolated.

1. First we prove that $m \geq 1$. Assume that $m = 0$. Then, $\neg l'(\bar{b}) \in IC_D$. One of the following cases must be true:
   (a) $\forall \bar{x} \neg l'(\bar{x}) \in IC$. In this case $\mathrm{T}_1(l'(\bar{x})) = l'(\bar{x}) \wedge \Box$. Thus $\mathrm{T}_1(l'(\bar{b})) = l'(\bar{b}) \wedge \Box$. Therefore, $r \not\models \mathrm{T}_1(l'(\bar{b}))$. A contradiction, because $r \models \mathrm{T}_\omega(l'(\bar{b}))$.
   (b) $\forall(\bar{x}, \bar{y})(\neg l'(\bar{x}) \vee \psi(\bar{x}, \bar{y})) \in IC$, where $\psi(\bar{x}, \bar{y})$ is a formula that only mentions built-in predicates. So $\mathrm{T}_1(l'(\bar{x})) = l'(\bar{x}) \wedge \forall \bar{y} \psi(\bar{x}, \bar{y})$. Therefore, $\mathrm{T}_1(l'(\bar{b})) = l'(\bar{b}) \wedge \forall \bar{y} \psi(\bar{b}, \bar{y})$. But $r \models \mathrm{T}_1(l'(\bar{b}))$, thus $r \models \forall \bar{y} \psi(\bar{b}, \bar{y})$. Then, we have that, for every tuple $\bar{c}$, $r \models \psi(\bar{b}, \bar{c})$. Thus, for every formula $\neg l'(\bar{b}) \vee \psi(\bar{b}, \bar{c})$ we have that $\psi(\bar{b}, \bar{c})$ is true in r; in consequence, by (a) and the definition of IC_D, $\neg l'(\bar{b}) \notin IC_D$; a contradiction.
2. Now, we prove that there exists $l_j(\bar{b}, \bar{a}_j) \in \Sigma_\omega$. First, notice that for $k \geq 1$, $T_k(l'(\bar{x}))$ is of the form:

$$l'(\bar{x}) \wedge \forall(\bigvee_{i=1}^m l_i(\bar{x}, \bar{x}_i) \wedge \mathrm{T}_{k-1}(l_i(\bar{x}, \bar{x}_i)) \vee \psi(\bar{x}, \bar{y})) \wedge \theta(\bar{x}).$$

Therefore, $T_k(l'(\bar{b}))$ is of the form

$$l'(\bar{b}) \wedge \forall(\bigvee_{i=1}^m l_i(\bar{b}, \bar{x}_i) \wedge \mathrm{T}_{k-1}(l_i(\bar{b}, \bar{x}_i)) \vee \psi(\bar{b}, \bar{y})) \wedge \theta(\bar{b}).$$

But $r \models \mathrm{T}_\omega(l'(\bar{b}))$, and then $r \models \mathrm{T}_k(l'(\bar{b}))$ for all $k \geq 1$. In consequence, for all $k - 1 \geq 0$,

$$r \models \bigvee_{i=1}^m l_i(\bar{b}, \bar{a}_i) \wedge \mathrm{T}_{k-1}(l_i(\bar{b}, \bar{a}_i)), \tag{6}$$

But, we know that, for every sentence α, if $r \nvDash \mathrm{T}_s(\alpha)$, then $r \nvDash \mathrm{T}_{s+1}(\alpha)$. Thus, by (6), there exists j ($1 \leq j \leq m$) such that $r \vDash \mathrm{T}_{k-1}(l_j(\bar{b}, \bar{a}_j))$ for all $k-1 \geq 0$. Therefore $r \vDash \mathrm{T}_\omega(l_j(\bar{b}, \bar{a}_j))$. We conclude that $l_j(\bar{b}, \bar{a}_j) \in \Sigma_\omega$ because $l_j(\bar{b}, \bar{a}_j) \in \Sigma_{n+1}$.

Now, we prove that $r^* \vDash IC_D$. Let θ be $\bigvee_{i=1}^k l_i(\bar{a}_i) \in IC_D$. We need to consider three cases:

1. If $l_j(\bar{a}_j) \in \Sigma_\omega$ for some j ($1 \leq j \leq k$), then $r^* \vDash l_j(\bar{a}_j)$. Thus, $r^* \vDash \theta$.
2. If $\neg l_j(\bar{a}_j) \in \Sigma_\omega$, then by (III.I) there exists $l_i(\bar{a}_i) \in \Sigma_\omega$, for some $1 \leq i \leq k$. Thus $r^* \vDash \theta$.
3. If for every i ($1 \leq i \leq m$), $l_i(\bar{a}_i) \notin \Sigma_\omega$ and $\neg l_i(\bar{a}_i) \notin \Sigma_\omega$, then for every m ($1 \leq i \leq m$), $r' \vDash l_i(\bar{a}_i)$ iff $r^* \vDash l_i(\bar{a}_i)$. Therefore, $r' \vDash \theta$ iff $r^* \vDash \theta$. But $r' \vDash IC$ and $\theta \in IC_D$, so we have that $r' \vDash \theta$. We conclude that $r^* \vDash \theta$.

Finally, we obtain that $r^* \vDash IC$, because $r^* \vDash IC_D$. We have a contradiction, because by (I), (II) and (III), we conclude that r' is not a repair of r, because $r^* \models IC$ and $\Delta(r, r^*) \subsetneq \Delta(r, r')$.

Lemma 2. If $r \vDash \mathrm{T}_\omega(\bigwedge_{i=1}^n l_i(\bar{a}_i))$, where $l_i(\bar{a}_i)$ is a ground literal, then for every repair r' of r, it holds $r' \vDash \bigwedge_{i=1}^n l_i(\bar{a}_i)$.

Proof: If $r \vDash \mathrm{T}_\omega(\bigwedge_{i=1}^n l_i(\bar{a}_i))$, then, for every $k \geq 0$ we have that $r \vDash \mathrm{T}_k(\bigwedge_{i=1}^n l_i(\bar{a}_i))$. But $T_k(\bigwedge_{i=1}^n l_i(\bar{a}_i)) = \bigwedge_{i=1}^n T_k(l_i(\bar{a}_i))$. Therefore, for every i and $k \geq 0$, $r \vDash \mathrm{T}_k(l_i(\bar{a}_i))$. Thus, for every i, $r \vDash \mathrm{T}_\omega(l_i(\bar{a}_i))$. By lemma 1, for every repair r' of r, $r' \vDash l_i(\bar{a}_i)$. Then, we conclude that $r' \vDash \bigwedge_{i=1}^n l_i(\bar{a}_i)$.

Lemma 3. If $r \vDash \mathrm{T}_\omega(\bigvee_{i=1}^n C_i(\bar{a}_i))$, with $C_i(\bar{a}_i)$ a conjunction of literals, then for every repair r' of r, $r' \vDash \bigvee_{i=1}^n C_i(\bar{a}_i)$.

Proof: If $r \vDash \mathrm{T}_\omega(\bigvee_{i=1}^n C_i(\bar{a}_i))$, then for every $k \geq 0$ we have that $r \vDash \mathrm{T}_k(\bigvee_{i=1}^n C_i(\bar{a}_i))$. We know that $T_k(\bigvee_{i=1}^n C_i(\bar{a}_i)) = \bigvee_{i=1}^n T_k(C_i(\bar{a}_i))$, and that for every sentence α, if $r \nvDash \mathrm{T}_s(\alpha)$, then $r \nvDash \mathrm{T}_{s+1}(\alpha)$. In consequence, there exists j ($0 \leq j \leq k$), such that, for all $k \geq 0$, $r \vDash \mathrm{T}_k(C_j(\bar{a}_j))$. Therefore $r \vDash \mathrm{T}_\omega(C_j(\bar{a}_j))$. By lemma 2 we have that for every repair r' of r, $r' \vDash C_j(\bar{a}_j)$. We conclude that $r' \vDash \bigvee_{i=1}^n C_j(\bar{a}_j)$.

Lemma 4. Let $Q(\bar{x})$ a universal query. If $r \vDash \mathrm{T}_\omega(Q(\bar{t}))$, for a ground tuple $\bar{t}$, then for every repair r' of r, $r' \vDash Q(\bar{t})$.

Proof: By induction on the number of universal quantifiers in $Q(\bar{x})$. If $Q(\bar{x})$ has zero quantifiers, then by lemma 3, $r' \vDash Q(\bar{t})$. Assume that the property is valid for every universal query with n quantifiers, and suppose that $Q(\bar{x})$ has $n+1$ universal quantifiers. Then, $Q(\bar{x})$ is of the form $\forall x Q'(x, \bar{x})$. If $r \vDash \mathrm{T}_\omega(Q(\bar{t}))$, then, for every $k \geq 0$, $r \vDash \mathrm{T}_k(\forall x Q'(x, \bar{t}))$. But $\mathrm{T}_k(\forall x Q'(x, \bar{t})) = \forall x \mathrm{T}_k(Q'(x, \bar{t}))$, thus $r \vDash \forall x \mathrm{T}_k(Q'(x, \bar{t}))$. In consequence, for every element a in the domain of the database, $r \vDash \mathrm{T}_k(Q'(a, \bar{t}))$, for every $k \geq 0$. Thus, for every element a in the database domain, $r \vDash \mathrm{T}_\omega(Q'(a, \bar{t}))$. By induction hypothesis we obtain that, for every repair r' of r and every element a in the database domain, $r' \vDash Q'(a, \bar{t})$. Therefore, for every repair r' of r, $r' \vDash Q(\bar{t})$.

Lemma 5. Let $Q(\bar{x})$ a domain independent query. If $r \vDash \mathrm{T}_\omega(Q(\bar{t}))$, for a ground tuple $\bar{t}$, then for every repair r' of r, $r' \vDash Q(\bar{t})$.

Proof: By induction on the number of quantifiers in $Q(\bar{x})$. If $Q(\bar{x})$ has zero quantifiers, then by lemma 3, $r' \vDash Q(\bar{t})$. Assume that the property is valid for every domain independent query with n quantifiers, and assume that $Q(\bar{x})$ has $n+1$ quantifiers. We need to consider two cases:

1. $Q(\bar{x})$ is of the form $\forall x Q'(x, \bar{x})$. If $r \vDash \mathrm{T}_\omega(Q(\bar{t}))$, then, for every $k \geq 0$, $r \vDash \mathrm{T}_k(\forall x Q'(x, \bar{t}))$. But $\mathrm{T}_k(\forall x Q'(x, \bar{t})) = \forall x \mathrm{T}_k(Q'(x, \bar{t}))$, then $r \vDash \forall x \mathrm{T}_k(Q'(x, \bar{t}))$. Therefore, for every element a in the domain of the database, $r \vDash \mathrm{T}_k(Q'(a, \bar{t}))$, for every $k \geq 0$. Thus for every element a in the database domain, $r \vDash \mathrm{T}_\omega(Q'(a, \bar{t}))$. By induction hypothesis we conclude that for every repair r' of r and every element a in the database domain, $r' \vDash Q'(a, \bar{t})$. In consequence, for every repair r' of r, $r' \vDash Q(\bar{t})$.
2. $Q(\bar{x})$ is of the form $\exists x Q'(x, \bar{x})$. If $Q(\bar{x})$ is a domain independent query then is possible to replace the existential quantifier by a finite disjunction of the form $\bigvee_{a \in dom(Q)}$. Then, it holds $r \vDash \mathrm{T}_\omega(\bigvee_{a \in dom(Q)} Q'(a, \bar{t}))$. Then, for every $k \geq 0$, $r \vDash \mathrm{T}_k(\bigvee_{a \in dom(Q)} Q'(a, \bar{t}))$. But $\mathrm{T}_k(\bigvee_{a \in dom(Q)} Q'(a, \bar{t})) = \bigvee_{a \in dom(Q)} \mathrm{T}_k(Q'(a, \bar{t}))$, and we know that for every sentence α, if $r \nvDash \mathrm{T}_s(\alpha)$, then $r \nvDash \mathrm{T}_{s+1}(\alpha)$. We conclude that there exists $b \in dom(Q)$, such that, for every $k \geq 0$, $r \vDash \mathrm{T}_k(Q'(b, \bar{t}))$. Therefore, $r \vDash \mathrm{T}_\omega(Q'(b, \bar{t}))$. By induction hypothesis, for every repair r' of r, we obtain that $r' \vDash Q'(b, \bar{t})$. Thus, for every repair r' of r, $r' \vDash \exists x Q'(x, \bar{t})$.

Proof of Theorem 2: Lemmas 4 and 5.

Proposition 2. Given a set IC of integrity constraints, a ground clause $\bigvee_{i=1}^m l_i(\bar{t}_i)$, if $IC \nvDash_{DB} \bigvee_{i=1}^m l_i(\bar{t}_i)$ and, for every repair r' of r, $r' \vDash \bigvee_{i=1}^m l_i(\bar{t}_i)$, then $r \vDash \bigvee_{i=1}^m l_i(\bar{t}_i)$. □

Proof of Proposition 2: Assume that $r \vDash \neg \bigvee_{i=1}^m l_i(\bar{t}_i)$. By hypothesis $IC \nvDash_{DB} \bigvee_{i=1}^m l_i(\bar{t}_i)$, thus there exists an instance of the database r' such that $r' \vDash IC \cup \{\neg \bigvee_{i=1}^m l_i(\bar{t}_i)\}$. Let us consider the set of database instances

$$R = \{r^* | r^* \vDash IC \text{ and } \Delta(r, r^*) \subseteq \Delta(r, r')\}.$$

We know that $\Delta(r, r')$ is finite, therefore there exists $r_0 \in R$ such that $\Delta(r, r_0)$ is minimal. Then, r_0 is a repair of r.

For every $1 \leq i \leq m$, if $l_i(\bar{t}_i)$ is $p(\bar{t})$ or $\neg p(\bar{t})$, then $p(\bar{t}) \notin \Delta(r, r')$. Using this fact we conclude that $p(\bar{t}) \notin \Delta(r, r_0)$, Therefore, $r \vDash \bigvee_{i=1}^m l_i(\bar{t}_i)$ if and only if $r_0 \vDash \bigvee_{i=1}^m l_i(\bar{t}_i)$. But we assumed that $r \vDash \neg \bigvee_{i=1}^m l_i(\bar{t}_i)$, then $r_0 \vDash \neg \bigvee_{i=1}^m l_i(\bar{t}_i)$; a contradiction.

Proof of Theorem 2: From the theorem 4.

Proof of Corollary 2: In this case it holds:

1. For every tuple $\bar{a}$, $IC \not\models_{DB} P_i(\bar{a})$, because the empty database instance (which has only empty base relations) satisfies IC, but not $P(\bar{a})$.
2. For every tuple $\bar{a}$, $IC \not\models_{DB} \neg P_i(\bar{a})$, since the database instance $r^i_{\bar{a}}$, where the relation P_i contains only the tuple $\bar{a}$ and the other relations are empty, satisfies IC, but not $\neg P_i(\bar{a})$.

Proof of Theorem 4: Suppose that $r \models_c l(\bar{t})$. Let r' a repair of r, we have that $r' \models l(\bar{t})$. By proposition 1 we have that $r' \models T_n(l(\bar{t}))$, that is

$$r' \models l(\bar{t}) \wedge \bigwedge_{i=1}^{m} \forall(\bigvee_{j=1}^{m_i} l_{i,j}(\bar{t},\bar{x}_{i,j}) \vee \psi_i(\bar{t},\bar{x}_i)), \tag{7}$$

We want to prove that for every i and for every sequence of ground tuples a_i, $a_{i,1}$, $\ldots$, a_{i,m_i}:

$$r \models \bigvee_{j=1}^{m_i} l_{i,j}(\bar{t},\bar{a}_{i,j}) \vee \psi_i(\bar{t},\bar{a}_i), \tag{8}$$

To do this, first we are going to prove that for every $i \in S$ and for every sequence of ground tuples a_i, $a_{i,1}$, $\ldots$, a_{i,m_i}:

$$r \models \bigvee_{j=1}^{m_i} l_{i,j}(\bar{t},\bar{a}_{i,j}) \vee \psi_i(\bar{t},\bar{a}_i), \tag{9}$$

This is immediately obtained when $r \models \psi_i(\bar{t},\bar{a}_i)$. Assume that $r \models \neg\psi_i(\bar{t},\bar{a}_i)$. We know that ψ_i only mentions built-in predicates, thus for every repair r' of r we have that $r' \models \neg\psi_i(\bar{t},\bar{a}_i)$. Therefore, by (7) we conclude that for every repair r' of r:

$$r' \models \bigvee_{j=1}^{m_i} l_{i,j}(\bar{t},\bar{a}_{i,j}) \vee \psi_i(\bar{t},\bar{a}_i),$$

By proposition 2 we conclude (9). Thus we have that

$$r \models l(\bar{t}) \wedge \bigwedge_{i \in S} \forall(\bigvee_{j=1}^{m_i} l_{i,j}(\bar{t},\bar{x}_{i,j}) \vee \psi_i(\bar{t},\bar{x}_i)),$$

but by the second condition in the hypothesis of the theorem we conclude that:

$$r \models l(\bar{t}) \wedge \bigwedge_{i=1}^{m} \forall(\bigvee_{j=1}^{m_i} l_{i,j}(\bar{t},\bar{x}_{i,j}) \vee \psi_i(\bar{t},\bar{x}_i)).$$

Proof of Theorem 5:
($\Longrightarrow$) Suppose that IC is acyclic, then there exists f as in the definition 14. We are going to prove by induction on k that for every literal name l, if $f(l) = k$, then $T_{k+1}(l(\bar{x})) = T_{k+2}(l(\bar{x}))$

(I) If $k = 0$. We know that that for every literal name l', $f(l') \geq 0$. Therefore, every integrity constraint containing $\neg l$ is of the form $\forall(\neg l(\bar{x}) \vee \psi(\bar{y}))$, where ψ only mentions built-in predicates. This is because if there were any other literal l' in the integrity constraint, we would have $f(l') < f(l) = 0$. Then $T_1(l(\bar{x})) = T_2(l(\bar{x}))$.

(II) Suppose that the property is true for every $m < k$. We know that $T_{k+2}(l(\bar{x}))$ is of the form:

$$l(\bar{x}) \wedge \bigwedge_{i=1}^{m} \bar{Q}_i(\bigvee_{j=1}^{m_i} T_{k+1}(l_{i,j}(\bar{x}_{i,j})) \vee \psi_i(\bar{x}_i)),$$

where $\bar{Q}_i$ is a sequence of quantifiers over all the variables $\bar{x}_{i,1}$, $\ldots$, $\bar{x}_{i,m_i}$, $\bar{x}_i$ not appearing in $\bar{x}$, and $T_{k+1}(l(\bar{x}))$ is of the form:

$$l(\bar{x}) \wedge \bigwedge_{i=1}^{m} \bar{Q}_i(\bigvee_{j=1}^{m_i} T_k(l_{i,j}(\bar{x}_{i,j})) \vee \psi_i(\bar{x}_i)).$$

By definition of f, we know that for every literal name $l_{i,j}$ in the previous formulas, $f(l_{i,j}) < k$. Then by induction hypothesis $T_k(l(\bar{x}_{i,j})) = T_{k+1}(l_{i,j}(\bar{x}_{i,j}))$ (since if $T_m(l'(\bar{x})) = T_{m+1}(l'(\bar{x}))$, then for every $n \geq m$, $T_n(l'(\bar{x})) = T_{n+1}(l'(\bar{x}))$).

($\Longleftarrow$) Suppose that for every literal name l, $\mathrm{T}_\omega(l(\bar{x}))$ is finite. The for every literal name l there exists a first natural number k such that $\mathrm{T}_k(l(\bar{x})) = \mathrm{T}_{k+1}(l(\bar{x}))$. Let us define a function f, from the literal names into the natural number, by $f(l) = k$ (k as before). We can show that this is a well defined function that behaves as in definition 14: since if $\forall(\bigvee_{i=1}^{m} l_i(\bar{x}_i) \vee \psi(\bar{y})) \in IC$, then for every $1 \leq s \leq m$, $\mathrm{T}_{f(\neg l_s)}(\neg l_s(\bar{x}_s))$ is of the form

$$\neg l_s(\bar{x}_s) \wedge \bar{Q}(\bigvee_{i=1}^{s-1} \mathrm{T}_{f(\neg l_s)-1}(l_i(\bar{x}_i)) \vee \bigvee_{i=s+1}^{m} \mathrm{T}_{f(\neg l_s)-1}(l_i(\bar{x}_i)) \vee \psi(\bar{y})) \wedge \theta(\bar{x}_s), \tag{10}$$

where $\bar{Q}$ is a sequence of quantifiers over all the variables $\bar{x}_1$, $\ldots$, $\bar{x}_m$, $\bar{y}$, not appearing in $\bar{x}_s$, and $\mathrm{T}_{f(\neg l_s)+1}(\neg l_s(\bar{x}_s))$ is of the form

$$\neg l_s(\bar{x}_s) \wedge \bar{Q}(\bigvee_{i=1}^{s-1} \mathrm{T}_{f(\neg l_s)}(l_i(\bar{x}_i)) \vee \bigvee_{i=s+1}^{m} \mathrm{T}_{f(\neg l_s)}(l_i(\bar{x}_i)) \vee \psi(\bar{y})) \wedge \theta'(\bar{x}_s). \tag{11}$$

By definition of f, $T_{f(\neg l_s)}(\neg l_s(\bar{x}_s)) = T_{f(\neg l_s)+1}(\neg l_s(\bar{x}_s))$. Then, by the form of (10) and (11), we conclude that for every

$i \neq s$, $T_{f(\neg l_s)-1}(l_i(\bar{x}_i)) = T_{\neg f(l_s)}(l_i(\bar{x}_i))$, and then, again by definition of f, $f(l_i) < f(\neg l_s)$.

Proof of Corollary 3: The following stratification function from literals to $\mathbb{N}$ can be defined: $f(\neg P_i) = 0$ and $f(P_j) = 1$, where P_i, P_j are relation names.

Lemma 6. If $\mathrm{T}_n(l(\bar{x}))$ is of the form:

$$l(\bar{x}) \wedge \bigwedge_{i=1}^{m} \forall(\bar{x}_i, \bar{y}_i)(C_i(\bar{x}, \bar{x}_i) \vee \psi_i(\bar{x}, \bar{y}_i)),$$

then $\mathrm{T}_{n+1}(l(\bar{x}))$ is of the form:

$$l(\bar{x}) \wedge \bigwedge_{i=1}^{m} \forall(\bar{x}_i, \bar{y}_i)(\mathrm{T}_1(C_i(\bar{x}, \bar{x}_i)) \vee \psi_i(\bar{x}, \bar{y}_i)),$$

Proof: From the definition of T_{n+1}.

Lemma 7. If for a ground tuple $\bar{a}$, $\mathrm{T}_n(l(\bar{a})) \vDash \forall(\bigvee_{j=1}^{k} l'_j(\bar{a}, \bar{z}_j))$, then $\mathrm{T}_{n+1}(l(\bar{a})) \vDash \forall(\bigvee_{j=1}^{k} \mathrm{T}_1(l'_j(\bar{a}, \bar{z}_j)))$.

Proof of Theorem 7: Suppose that for a natural number n, $\forall\bar{x}(\mathrm{T}_n(l(\bar{x})) \supset \mathrm{T}_{n+1}(l(\bar{x})))$ is a valid sentence. We are going to prove that for every $m \geq n$, $\forall\bar{x}(\mathrm{T}_m(l(\bar{x})) \supset \mathrm{T}_{m+1}(l(\bar{x})))$ is a valid sentence, by induction on m.

(I) If $m = n$, by hypothesis.

(II) Suppose that $\forall\bar{x}(\mathrm{T}_m(l(\bar{x})) \supset \mathrm{T}_{m+1}(l(\bar{x})))$ is a valid sentence. For every clause $\bigvee_{j=1}^{k} l'_j(\bar{x}, \bar{z}_j) \vee \psi(\bar{x}, \bar{z})$ in $\mathrm{T}_{m+1}(l(\bar{x}))$ and for every ground tuple $\bar{a}$ we have that

$$\mathrm{T}_m(l(\bar{a})) \vDash \forall(\bigvee_{j=1}^{k} l'_j(\bar{a}, \bar{z}_j) \vee \psi(\bar{a}, \bar{z})).$$

By lemma 7 and considering that ψ only mentions built-in predicates we have that $\mathrm{T}_{m+1}(l(\bar{a})) \vDash \forall(\bigvee_{j=1}^{k} \mathrm{T}_1(l'_j(\bar{a}, \bar{z}_j)) \vee \psi(\bar{a}, \bar{z}))$, and from this and lemma 6 we can conclude that $\forall\bar{x}(\mathrm{T}_{m+1}(l(\bar{x})) \supset \mathrm{T}_{m+2}(l(\bar{x})))$ is a valid sentence.

Proof of Theorem 6: For uniform constraints the residues do not contain quantifiers. Therefore $\mathrm{T}_n(l(\bar{x}))$ for every $n \geq 0$ is quantifier-free and contains only the variables that occur in $\bar{x}$. There are only finitely many inequivalent formulas with this property, and thus $\mathrm{T}_\omega(l(\bar{x}))$ is finite.

Proof of Theorem 8: Let $Q(\bar{x})$ be a domain independent query and r a database instance. Define $A_n = \{\bar{t} \mid r \vDash \mathrm{T}_n(Q(\bar{t}))\}$. We know that for every n: $A_{n+1} \subseteq A_n$, therefore $A = \{A_i \mid i < \omega\}$ is a family of subsets of A_0. But A_0 is finite because $Q(\bar{x})$ is a domain independent query. Thus, there exists a minimal element A_m in A. For this element, it holds that for every $k \geq m$: $A_m = A_k$, since $A_k \subseteq A_m$.