Assignment #4 Due on date of Second Midterm, Tuesday April 8th

1. In the figure on the next page find the disparity d for the point P located at (10, 20, 10). Use the fact that $d = f\,T / Z$.

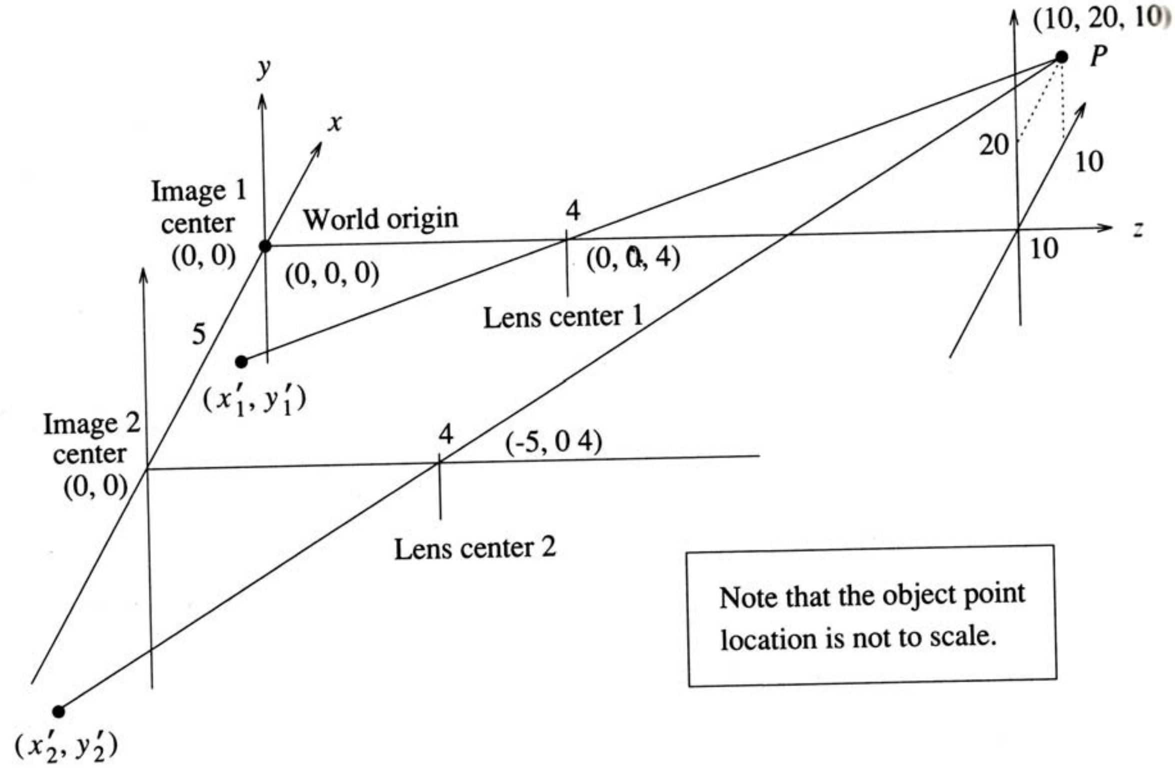
2. Consider two points A and B in a simple stereo system. Point A projects to Al on the left image, and Ar on the right image. Similarly there is a point B which projects to Bl and Br. Consider the order of these two points in each image on their epipolar lines. There are two possibilities; either they ordered on the epipolar lines in the same order; for example they appear as Al, Bl and Ar Br, or they are in opposite order, such as Bl, Al and Ar,Br. Place the two 3d points A and B in two different locations in a simple stereo diagram which demonstrates these two possibilities. (Draw a different picture for each situation).

3. The equation of a simple stereo system is $z = f\,T / d$. In this question assume that $f\,T = 1$ which means that $z = 1 / d$. Also you can assume that the only source of error in a simple stereo system is the error in estimating the disparity, and that this error is exactly one pixel. So if the stereo system says the disparity is 5 pixels it is really between 4 and 6 pixels. And if the stereo system says the disparity is 10 pixels then it is really between 9 and 11 pixels. The change in Z (DeltaZ) due to this one pixel error in estimating the disparity is called the absolute error of the stereo system. By this I mean that $\text{DeltaZ}(X \text{ pixels}) = \| z(X-1) - z(X+1) \|$. Compute the ratio of DeltaZ(5 pixels) /DeltaZ(10 pixels). From this answer hypothesize a relationship between DeltaZ and disparity d. Prove that your hypothesis is true by computing the derivative of Z with respect to disparity d.

4. There is a simple stereo system with one camera is placed above the other in the y direction (not the x direction is as usual) by a distance of b. In such a case there is no rotation between the

cameras, only a translation by a vector $T = [0,b,0]$. First compute the essential matrix E in this case. Assume that both cameras have the same focal length f. Prove that in this situation, for the computed E, that the epipolar lines are vertical. To do this it is enough to prove that for a given point (p_b) in the bottom image that the epipolar line in the top image defined by the equation $(p_t)^T E\ (p_b) = 0$ is a vertical line. Here p_t is (x_t,y_t,f) and p_b is (x_b,y_b,f) which are the matching points in the top and bottom image plane. You need to write out the equation of the line which contains p_t (the free variable) when you are given p_b and E.

5. Assume that there is a 3D point X on a plane that is viewed by two cameras. The projection of this 3D point in camera one is defined by $x = P X$, and in camera two by $x' = P' X$. Here $x = [u,v,1]$ the pixel co-ordinates in image one of X and $x' = [u',v',1]$ the pixel co-ordinates in the other images, P and P' are the 3 by 4 projection matrices and X is a point in 3D space = $[x,y,z,1]$. Prove that in this case (when the point X is on a plane) $x = M x'$, where M is a 3 by 3 matrix called a homography. Hint: Define the world co-ordinate frame for the 3D point X so that the x, y axis is on the plane containing the point X. In this case when X is a point on the plane it implies that X is defined as $X = [x,y,0,1]$ in homogeneous co-ordinates. Now write down the two projection equations for this point X and make use of the fact this same point X is seen by both cameras, each of which has a different viewpoint.


(10, 20, 10)
P
y
x
20
10
Image 1
center
(0, 0)
World origin
4
z
(0, 0, 0)
(0, 0, 4)
10
Lens center 1
5
(x'_1, y'_1)
Image 2
center
(0, 0)
4
(-5, 0 4)
Lens center 2
Note that the object point
location is not to scale.
(x'_2, y'_2)