# Geometric Model of Camera

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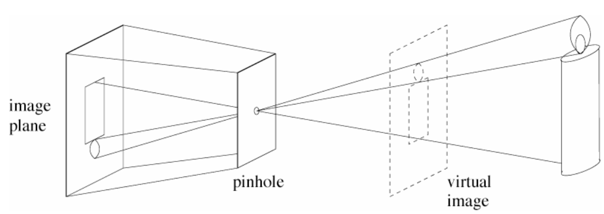
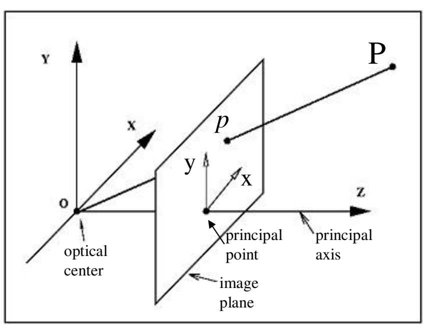
## Geometric Model of Camera

Perspective projection

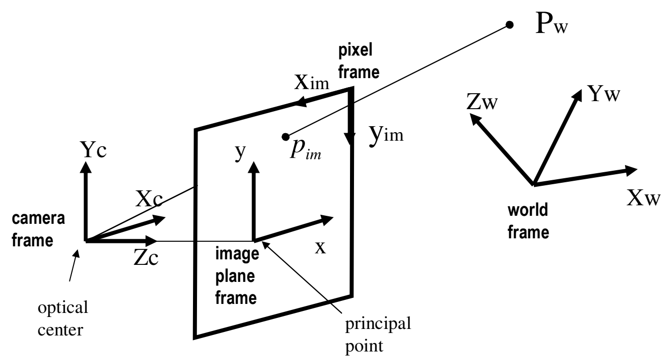



P(X,Y,Z) $\longrightarrow$ *p*(x,y)

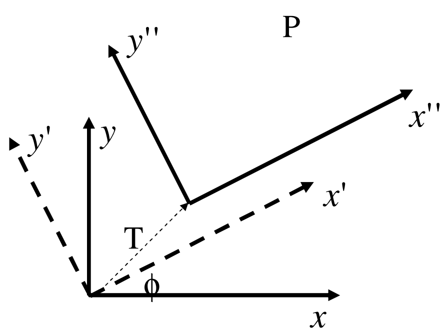
$$x = f\,\frac{X}{Z} \quad y = f\,\frac{Y}{Z}$$

## Four Coordinate Frames




Camera model: $$p_{im} = \begin{bmatrix} \text{transformation} \\ \text{matrix} \end{bmatrix} P_w$$

## Coordinate Transformation – 2D




Rotation and Translation

$$p' = \begin{bmatrix} p_x' \\ p_y' \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$p'' = \begin{bmatrix} p_x' \\ p_y' \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

## Homogeneous Coordinates

Go one dimensional higher:

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

$w$ is an arbitrary non-zero scalar, usually we choose 1.

From homogeneous coordinates to Cartesian coordinates:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 / x_3 \\ x_2 / x_3 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 / x_4 \\ x_2 / x_4 \\ x_3 / x_4 \end{bmatrix}$$

## 2D Transformation with Homogeneous Coordinates

2D coordinate transformation:

$$p'' = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

2D coordinate transformation using homogeneous coordinates:

$$\begin{bmatrix} p_x'' \\ p_y'' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & T_x \\ -\sin\phi & \cos\phi & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

## 3D Rotation Matrix

Rotate around each coordinate axis:

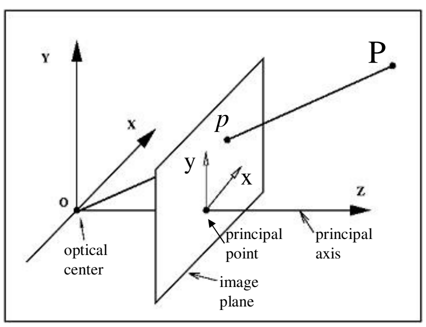
$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \quad R_2(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad R_3(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Combine the three rotations:

$$R = R_1 R_2 R_3$$

3D rotation matrix has three parameters.

## Perspective Projection




$$x = f\frac{X}{Z} \quad y = f\frac{Y}{Z}$$

These are *nonlinear*.

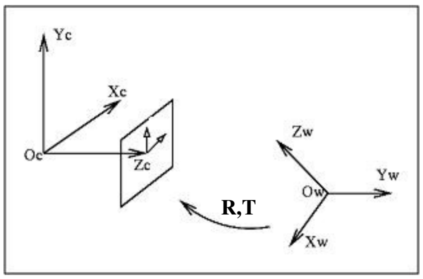
Using homogenous coordinate, we have a *linear* relation:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = u / w \quad y = v / w$$

## World to Camera Coordinate

Transformation between the camera and world coordinates:



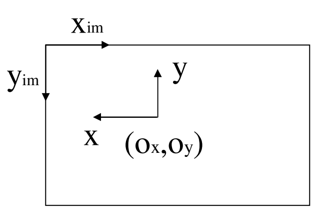

$$\mathbf{X}_c = \mathbf{R}\mathbf{X}_w + \mathbf{T}$$

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{T} \\ \mathbf{0}^{\mathrm{T}} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

## Image Coordinates to Pixel Coordinates

$$x = (o_x - x_{im})s_x \quad y = (o_y - y_{im})s_y$$

$s_x, s_y$ : pixel sizes




$$\begin{bmatrix} x_{im} \\ y_{im} \\ 1 \end{bmatrix} = \begin{bmatrix} -1/s_x & 0 & o_x \\ 0 & -1/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

## Put All Together – World to Pixel

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1/s_x & 0 & o_x \\ 0 & -1/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$= \begin{bmatrix} -1/s_x & 0 & o_x \\ 0 & -1/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/s_x & 0 & o_x \\ 0 & -1/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = K[R \quad T] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\boxed{x_{im} = x_1 / x_3 \quad y_{im} = x_2 / x_3}$$

## Camera Intrinsic Parameters

$$K = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

K is a 3x3 upper triangular matrix, called the **Camera Calibration Matrix**.

There are five intrinsic parameters:
(a) The pixel sizes in x and y directions $s_x, s_y$
(b) The focal length $f$
(c) The principal point ($o_x$,$o_y$), which is the point where the optic axis intersects the image plane.

## Extrinsic Parameters

$$p_{im} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = K\begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = M \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

[R|T] defines the **extrinsic parameters**.
The 3x4 matrix M = K[R|T] is called the **projection matrix**.

## Weak Perspective Model

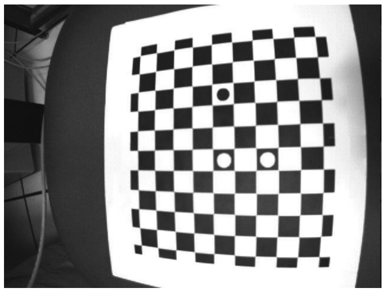
If the relative distance between any two points along the principal axis is much smaller than the average distance $\bar{z}$
The camera projection can be approximated as:

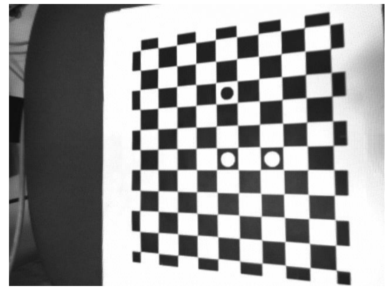
$$x = f\frac{X}{Z} \approx \frac{f}{\bar{Z}}X$$

$$y = f\frac{Y}{Z} \approx \frac{f}{\bar{Z}}Y$$

This is the **weak-perspective** camera model.

## Radial Distortions





$$x = x_d (1 + k_1 r^2 + k_2 r^4)$$
$$y = y_d (1 + k_1 r^2 + k_2 r^4)$$

$(x_d, y_d)$ are the coordinates of the distorted points, and

$r^2 = x_d^2 + y_d^2$