Filtering (II)

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COMP 4900C
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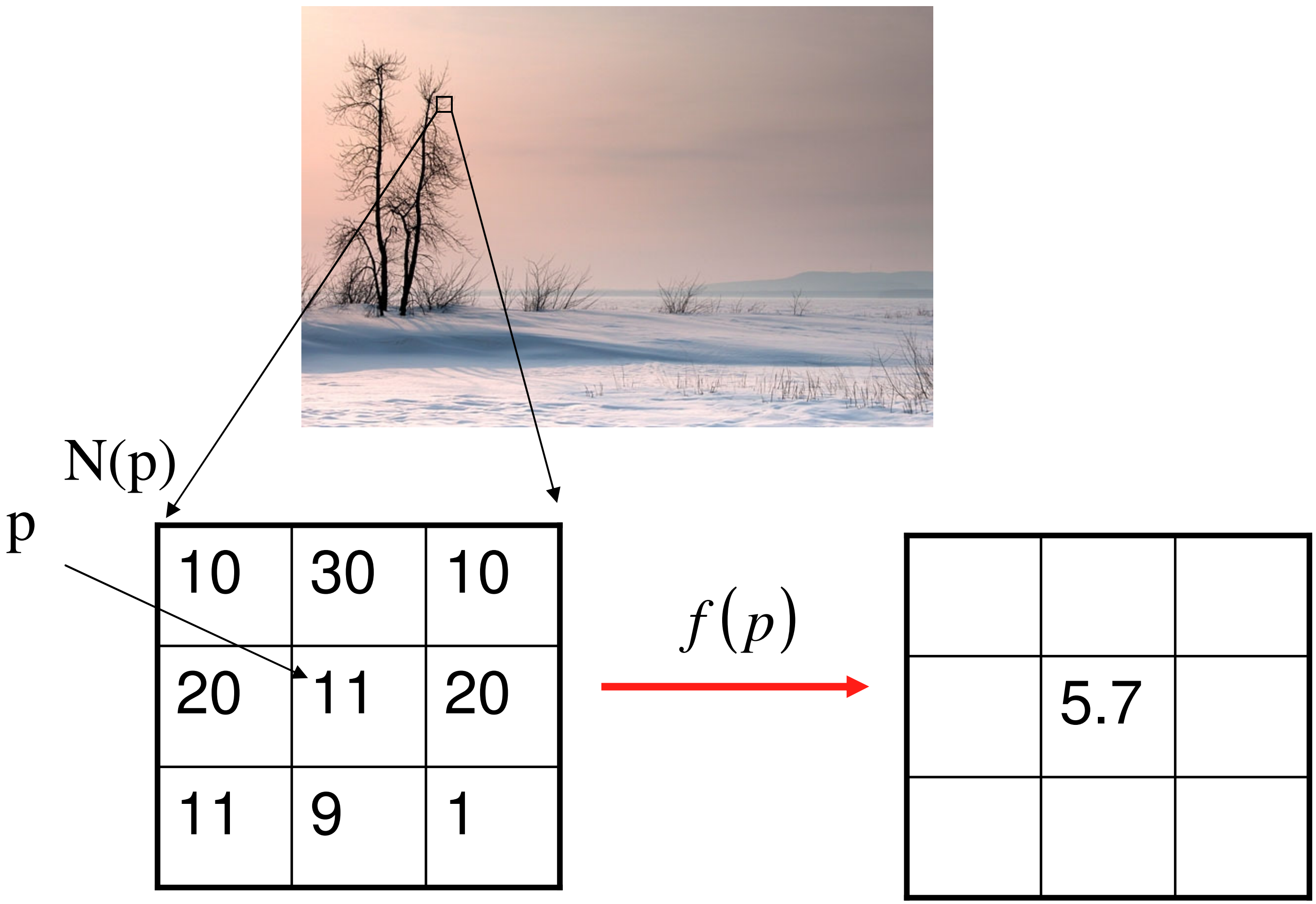
Image Filtering

Modifying the pixels in an image based on some functions of a local neighbourhood of the pixels



Linear Filtering – convolution

The output is the linear combination of the neighbourhood pixels

$$I_A(i,j) = I * A = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} A(h,k) I(i-h, j-k)$$

The coefficients come from a constant matrix A, called kernel.
This process, denoted by '*', is called (discrete) convolution.

Image

1	3	0
2	10	2
4	1	1

*

Kernel

1	0	-1
1	0.1	-1
1	0	-1

=

Filter Output

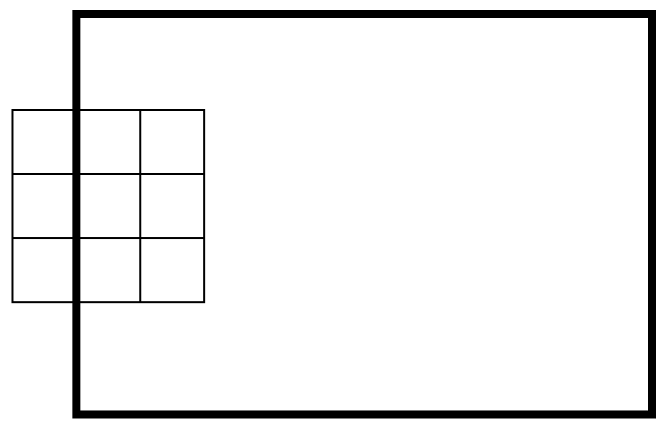
	5	

Handle Border Pixels

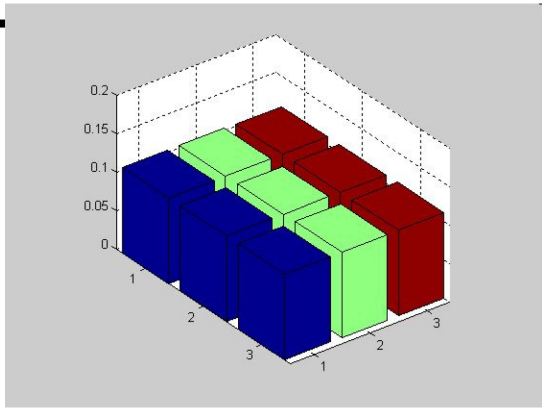
Near the borders of the image, some pixels do not have enough neighbours. Two possible solutions are:

- Set the value of all non-included pixels to zero.

- Set all non-included pixels to the value of the corresponding pixel in the input image.



Smoothing by Averaging

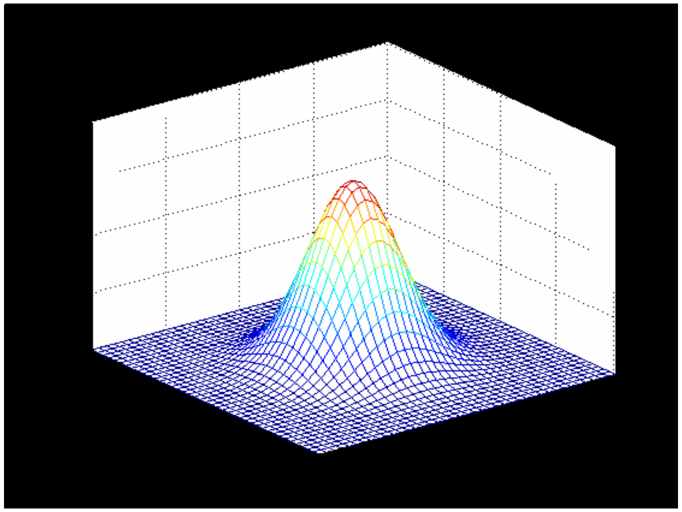


$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

Convolution can be understood as weighted averaging.

Gaussian Filter

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2 + y^2)}{2\sigma^2} \right)$$

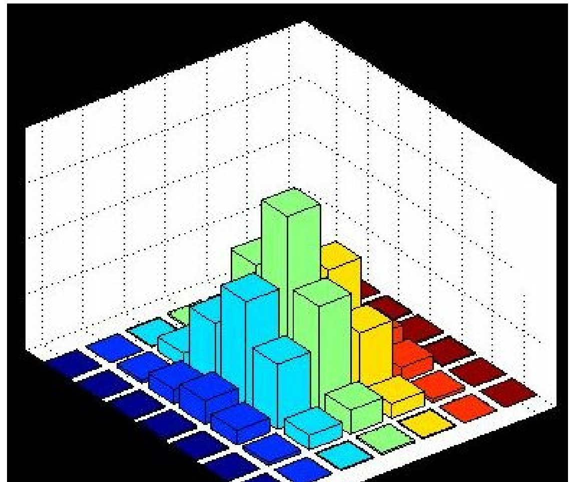


Discrete Gaussian kernel:

$$G(h, k) = \frac{1}{2\pi\sigma^2} e^{-\frac{h^2+k^2}{2\sigma^2}}$$

where $G(h, k)$ is an element of an $\mathrm{m} \times \mathrm{m}$ array

Gaussian Filter



$* \ \frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

$=$

$$\sigma = 1$$

Gaussian Kernel is Separable

$$I_G = I * G =$$

$$= \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} G(h,k) I(i-h, j-k) =$$

$$= \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} e^{-\frac{h^2+k^2}{2\sigma^2}} I(i-h, j-k) =$$

$$= \sum_{h=-m/2}^{m/2} e^{-\frac{h^2}{2\sigma^2}} \sum_{k=-m/2}^{m/2} e^{-\frac{k^2}{2\sigma^2}} I(i-h, j-k)$$

since $$e^{-\frac{h^2+k^2}{2\sigma^2}} = e^{-\frac{h^2}{2\sigma^2}} e^{-\frac{k^2}{2\sigma^2}}$$

Gaussian Kernel is Separable

Convolving rows and then columns with a 1-D Gaussian kernel.

$$\boldsymbol{I} * \frac{1}{38}\begin{bmatrix} 1 & 9 & 18 & 9 & 1 \end{bmatrix} = \boldsymbol{I_r}$$

$$\boldsymbol{I_r} * \frac{1}{38}\begin{bmatrix} 1 \\ 9 \\ 18 \\ 9 \\ 1 \end{bmatrix} = \text{result}$$

The complexity increases linearly with m instead of with m^2.

Gaussian vs. Average

Gaussian Smoothing

Smoothing by Averaging

Noise Filtering

Gaussian Noise

After Averaging

After Gaussian Smoothing

Noise Filtering

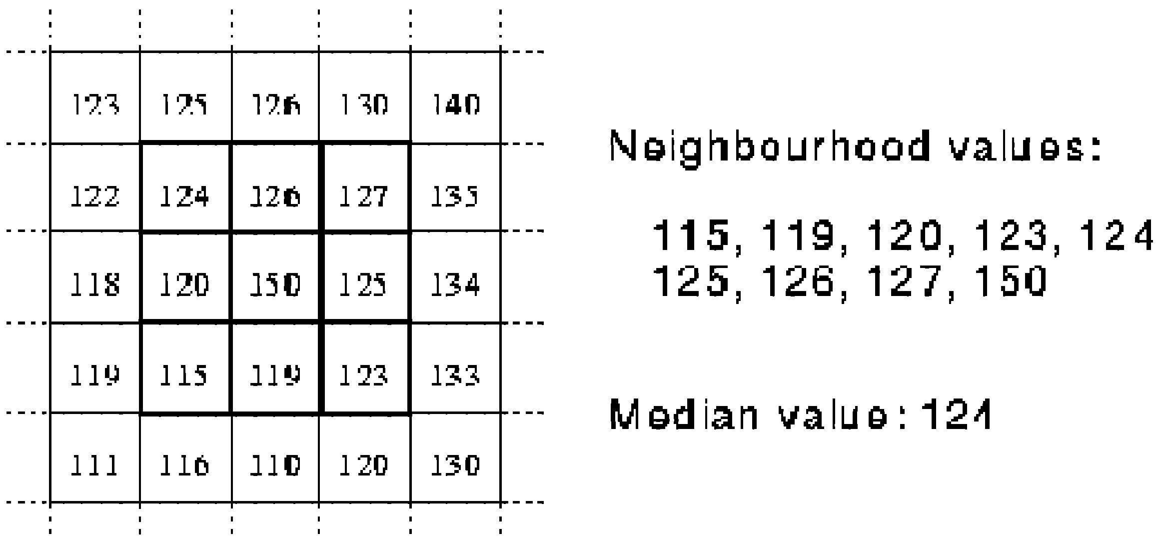
Salt-and-pepper noise

After averaging

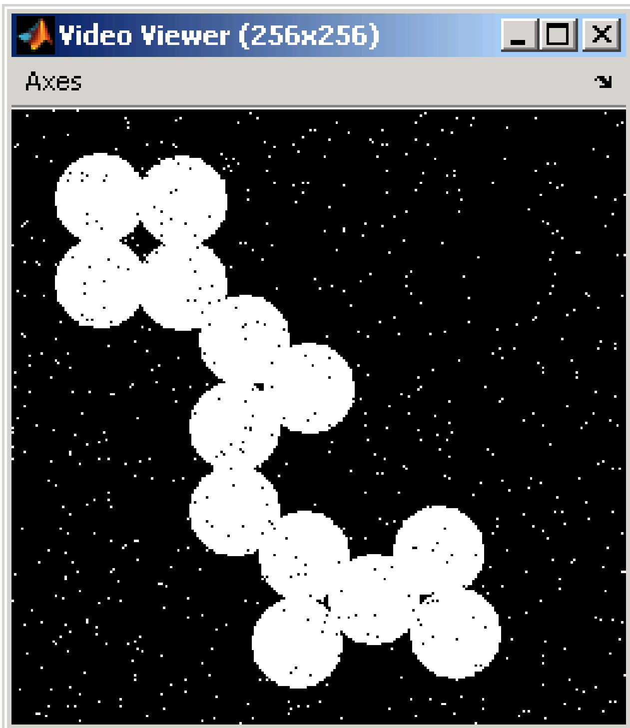
After Gaussian smoothing

Nonlinear Filtering – median filter

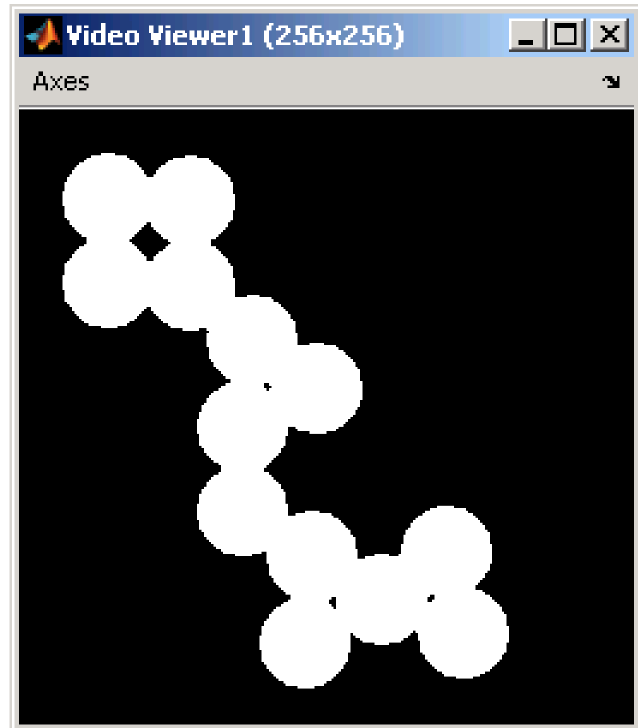
Replace each pixel value $I(i, j)$ with the median of the values found in a local neighbourhood of (i, j).



Median Filter



Salt-and-pepper noise



After median filtering

Salt-and-Pepper Noise Removal by Median-type Noise Detectors and Edge-preserving Regularization

Raymond H. Chan, Chung-Wa Ho, and Mila Nikolova

IEEE Transactions on Image Processing, 14 (2005), 1479-1485.