

# Parallel ROLAP Data Cube Construction On Shared-Nothing Multiprocessors \*

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## Abstract

*The pre-computation of data cubes is critical to improving the response time of On-Line Analytical Processing (OLAP) systems and can be instrumental in accelerating data mining tasks in large data warehouses. In order to meet the need for improved performance created by growing data sizes, parallel solutions for generating the data cube are becoming increasingly important. This paper presents a parallel method for generating data cubes on a shared-nothing multiprocessor. Since no (expensive) shared disk is required, our method can be used on low cost Beowulf style clusters consisting of standard PCs with local disks connected via a data switch. Our approach uses a ROLAP representation of the data cube where views are stored as relational tables. This allows for tight integration with current relational database technology.*

*We have implemented our parallel shared-nothing data cube generation method and evaluated it on a PC cluster, exploring relative speedup, local vs. global schedule trees, data skew, cardinality of dimensions, data dimensionality, and balance tradeoffs. For an input data set of 2,000,000 rows (72 Megabytes), our parallel data cube generation method achieves close to optimal speedup; generating a full data cube of  $\approx 227$  million rows (5.6 Gigabytes) on a 16 processors cluster in under 6 minutes. For an input data set of 10,000,000 rows (360 Megabytes), our parallel method, running on a 16 processor PC cluster, created a data cube consisting of  $\approx 846$  million rows (21.7 Gigabytes) in under 47 minutes.*

## 1 Introduction

The pre-computation of the different views (group-bys) of a data cube, i.e. the forming of aggregates for every combination of GROUP-BY attributes, is critical to improving the response time of On-Line Analytical Processing (OLAP) queries in decision support systems [10] and can be instrumental in accelerating data mining tasks in large data warehouses [11]. For a given raw data set,  $R$ , with  $n$  records and  $d$  attributes (dimensions), a view is constructed by an

aggregation of  $R$  along a subset of attributes. This results in  $2^d$  different possible views. Figure 1a shows the different possible view identifiers for 4 dimensions “A”, “B”, “C”, and “D”. An edge between two view identifiers indicates that one of the respective views can be computed from the other by aggregation along one dimension. The resulting graph is called the *lattice*. As proposed in [10], the pre-computation of the entire data cube (the set of all  $2^d$  views) allows for the fast execution of subsequent OLAP queries.

There are two basic data cube representations: ROLAP representations where views are represented as relational tables and MOLAP representations where views are represented as multi-dimensional arrays. Many methods have been proposed for generating the data cube on sequential [2, 12, 19, 20, 24, 25] and parallel systems [3, 5, 7, 8, 15, 16, 18]. The size of the data cube is potentially very large. In order to meet the need for improved performance created by growing data sizes in OLAP applications, parallel solutions for generating the data cube have become increasingly important. The current parallel approaches can be grouped into two broad categories: (1) *work partitioning* [3, 5, 15, 16, 18] and (2) *data partitioning* [7, 8].

Work partitioning methods assign different view computations to different processors. Consider, for example, the lattice for a four dimensional data cube as shown in Figure 1a. From the raw data set “ABCD”, 15 views need to be computed. Given a parallel computer with  $p$  processors, work partitioning schemes partition the set of views into  $p$  groups and assign the computation of the views in each group to a different processor. The main challenges for these methods are load balancing and scalability, which are addressed in different ways by the different techniques studied in [3, 5, 15, 18, 16]. One distinguishing feature of work partitioning methods is that all processors need simultaneous access to the entire raw data set. This access is usually provided through the use of a shared disk system (available e.g. for SunFire 6800 and IBM SP systems).

Data partitioning methods partition the raw data set into  $p$  subsets and store each subset locally on one processor. All views are computed on every processor but only with respect to the subset of data available at each processor. A

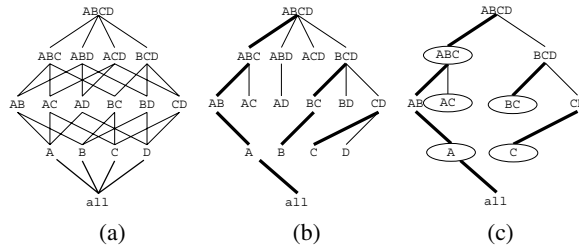
subsequent “merge” procedure is required to agglomerate data across processors. The advantage of data partitioning methods is that they do not require all processors to have access to the entire raw data set. Each processor only requires a local copy of a portion of the raw data which can, e.g., be stored on its local disk. This makes such methods feasible for shared-nothing parallel machines like the popular, low cost, Beowulf style clusters consisting of standard PCs connected via a data switch and without any (expensive) shared disk array; see Figure 2a. The main problem with data partitioning methods is that the “merge”, which has to be performed for every view of the data cube, has the potential to create massive data movements between processors with serious consequences for performance and scalability of the entire system. A data partitioning method for MOLAP representations has been presented in [7, 8]. This method is based on a space partitioning of the multi-dimensional array and a spatial “merge” between different sub-cubes of the MOLAP cube. The spatial “merge” operation can be reduced to a parallel prefix which is a well studied operation for parallel computers.

In this paper, we study data partitioning methods for the ROLAP case where the raw data set is given as a  $d$ -dimensional relation (table of  $d$ -tuples) and all views are to be created as relational tables as well. The principal advantage of ROLAP is that it allows for tight integration with current relational database technology. Another advantage of ROLAP is that it requires only linear space and is therefore particularly suitable for the construction of very large data cubes. Our algorithm is, to our knowledge, the first parallel ROLAP data cube construction method for shared-nothing multiprocessors. Our method has the additional advantage that it can be extended to the *partial* cube case where not all views but only a subset of views, selected by the user, are to be created. This case occurs frequently in practice because the user often knows that some views will not be required for on-line analytical processing (OLAP) queries on a given data set.

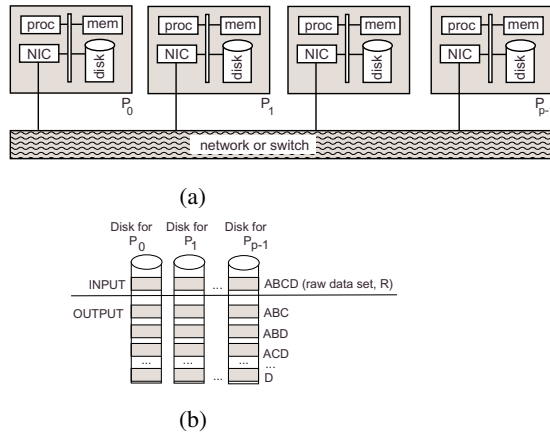
We have implemented our parallel data cube generation method and extensively evaluated it on a shared-nothing PC cluster. Our experiments have explored the following six performance issues: relative speedup, local vs. global schedule trees, data skew, cardinality of dimensions, data dimensionality, and balance tradeoffs. For a raw data set  $R$  of size  $n = 2,000,000$  rows (72 Megabytes) our parallel ROLAP data cube generation method achieved close to optimal speedup; generating a full data cube of  $\approx 227$  million rows (5.6 Gigabytes) in under 6 minutes on 16 processors. For an input data set of  $n = 10,000,000$  rows (360 Megabytes), our parallel method created the corresponding data cube consisting of  $\approx 846$  million rows (21.7 Gigabytes) in under 47 minutes.

The remainder of this paper is organized as follows. In Section 2 we present our parallel ROLAP data cube algorithm for shared-nothing multiprocessors. Section 3 shows

how our method can be extended to the partial cube case where not all views but only a subset of selected views are to be created. Section 4 discusses our implementation and presents the performance results achieved by our method. Section 5 concludes the paper.



**Figure 1. (a) Lattice for 4 dimensions “A”, “B”, “C”, and “D”. (b) Schedule tree for a full data cube (e.g. Pipesort). Bold edges represent “scan” operations and regular edges represent “sort” operations. (c) Schedule tree for a partial data cube. Selected views are marked by circles.**



**Figure 2. (a) A shared-nothing multiprocessor. (b) Input and output data distribution.**

## 2 A Parallel Data Cube Algorithm For Shared-Nothing Multiprocessors

We consider a shared-nothing parallel machine consisting of  $p$  processors  $P_0, P_1 \dots P_{p-1}$ , each with its own local memory and local disk, connected via a network or switch; see Figure 2a. There is no shared memory or shared disk available. Systems of this type include the popular, low cost, Beowulf style clusters consisting of standard PCs

connected via a switch (<http://www.beowulf.org/>). As input, we assume a raw data set,  $R$ , with  $n$  records and  $d$  dimensions  $D_0, D_1 \dots D_{d-1}$  distributed evenly over the  $p$  disks; see Figure 2b. The basic communication operation used by our data cube algorithm is the  $h$ -relation (method `MPI_ALL_TO_ALL_V` in MPI). Our method uses two basic local disk operations, applied by each processor to its local disk: (1) linear scan and (2) external memory sort [22]. For a processor  $P_j$  with local memory size  $m$  and a local disk with block transfer size  $B$ , a linear scan through a file of size  $n$  stored on its disk requires  $O(\frac{n}{B})$  block transfers between disk and memory while an external memory sort of that file requires  $O(\frac{n}{B} \log \frac{m}{B} \frac{n}{B})$  block transfers [22]. We will present our method for a shared-nothing multiprocessor with one local disk per processor  $P_j$ . However, it is easy to generalize our methods for machines with multiple local disks per processor by applying the linear scan and external memory sort methods for a single processor with multiple local disks presented in [23].

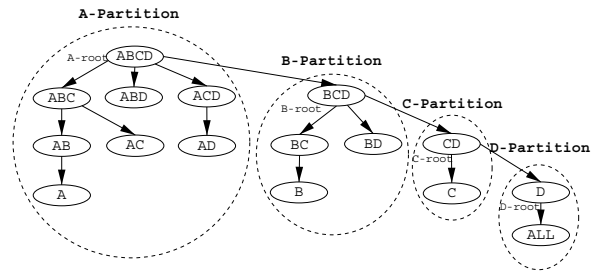
Without loss of generality, let  $|D_0| \geq |D_1| \geq \dots \geq |D_{d-1}|$ , where  $|D_i|$  is the cardinality for dimension  $D_i$ ,  $0 \leq i \leq d-1$  (i.e. the number of distinct values for dimension  $D_i$ ). Let  $S$  be the set of all  $2^d$  view identifiers. Each view identifier consists of a subset of  $\{D_0, D_1 \dots D_{d-1}\}$ , ordered by the cardinalities of the selected dimensions (in decreasing order). The goal is to create a data cube  $DC$  containing the views in  $S$ . We assume that, when our algorithm terminates, every view is distributed evenly across the  $p$  disks; see Figure 2b. It is important to note that, for the subsequent use of the views by OLAP queries, each view needs to be evenly distributed in order to achieve maximum IO bandwidth for subsequent parallel disk accesses.

## 2.1 Algorithm Outline

Our parallel algorithm uses as a building block a standard sequential top-down data cube method such as Pipesort [20]. Such methods have in common that they consist of a two phase approach. In the first phase, a *schedule tree*  $T$  is constructed which is a subgraph of the lattice and contains as nodes the identifiers of all views to be constructed. Recall that view  $v$  is a parent of a view  $v'$  if  $v$  can be created from  $v'$ . The schedule tree  $T$  identifies a sequence in which the views are to be constructed in the second phase. The main difference between the various top-down data cube methods is the schedule tree  $T$  that they build. For example, Pipesort starts with the lattice and assigns to every view identifier an estimate of the size of the respective view [6, 21]. It then computes the cost of the aggregate operation associated with each edge of the lattice. The schedule tree  $T$  is then built by scanning the lattice level by level, starting at the raw data set, and computing for each two subsequent levels of nodes, and the edges between them, a minimum cost bi-partite matching.

Let  $S_i \subset S$  be the subset of view identifiers in  $S$  that

start with  $D_i$ , and let  $DC_i$  be the data cube for  $S_i$ . We call  $DC_i$  the  $D_i$ -partition of the data cube  $DC$ . Furthermore, we refer to the view consisting of all dimensions contained in views of  $S_i$  as the  $D_i$ -root; see Figure 3.



**Figure 3. Partitions of a data cube for  $d = 4$ . Dimensions are labelled “A”, “B”, “C”, “D”,**

The following describes the global structure of our parallel data cube algorithm for shared-nothing multiprocessors. The algorithm consists of three main phases: data partitioning, computation of local  $D_i$ -partitions, and merge of local  $D_i$ -partitions. Subsequent sections will discuss each phase in more detail.

**Procedure 1 Parallel-Shared-Nothing-Data-Cube**  
 /\* Input: Raw data set  $R$  ( $n$   $d$ -dimensional records) distributed arbitrarily over the  $p$  processors,  $n/p$  records per processor; Output: Data cube,  $DC$ , distributed over the  $p$  processors. Each view is evenly distributed over the  $p$  processors' disks. \*/

FOR  $i=0$  TO  $d-1$

(1) /\* Data Partitioning \*/

(a) Each processor  $P_j$  ( $j = 0, \dots, p-1$ ) computes locally the  $D_i$ -root for its subset of data. (Essentially a sequential sort followed by a sequential scan.) Let  $D_i\text{-root}|_j$  denote the  $D_i$ -root created by processor  $P_j$ .

(b) /\* Sort  $\cup_{j=0, \dots, p-1} D_i\text{-root}|_j$  by  $D_i, \dots, D_{d-1}$  \*/

Adaptive-Sample-Sort( $D_i\text{-root}|_0, \dots, D_i\text{-root}|_{p-1}; D_i, \dots, D_{d-1}; \gamma = 1\%$ )

(c) Each processor  $P_j$  ( $j = 0, \dots, p-1$ ) computes locally the  $D_i$ -root for its subset of data received in the previous step. Let  $D_i\text{-root}|_j$  denote the  $D_i$ -root created by processor  $P_j$ .

(2) /\* Computation Of Local  $D_i$ -Partitions \*/

(a) Processor  $P_0$  locally computes, by applying the first phase of a sequential top-down data cube method, the schedule tree  $T_i$  for building the  $D_i$ -partition with respect to  $D_i\text{-root}|_0$ .

(b) Processor  $P_0$  broadcasts  $T_i$  to  $P_1 \dots P_{p-1}$ .

(c) Each processor  $P_j$  ( $j = 0, \dots, p-1$ ) computes locally the  $D_i$ -partition with respect to

$D_i$ -root $||_j$  by applying the second phase of a sequential top-down data cube method to the schedule tree  $T_i$  received in the previous step.

(3) /\* Merge Of Local  $D_i$ -Partitions \*/  
Merge-Partitions( $D_i$ )

The following Sections 2.2, 2.3 and 2.4 discuss in detail the three phases of Procedure 1.

## 2.2 Data Partitioning

Good data partitioning is a key factor in obtaining a good load balance and, consequently, good performance. Some researchers partition data on one or several dimensions [17, 9]. In order to achieve sufficient parallelism, they assume that the product of the cardinalities of these dimensions is much larger than the number of processors [9]. The advantage of their method is that they do not need to merge views. For examples, if we partition on  $A$ , then  $ABC$  and  $AC$  need not to be merged, or if we partition on  $A$  and  $B$ , then  $ABC$  and  $ABD$  need not to be merged. However, in practice, this assumption is often not true. The cardinality of some dimensions may be small, such as gender, months and intervals for a numeric attribute. The number of processors in a cluster may be large, especially for clusters of workstations. Therefore, those methods are often not scalable. Our method avoids these problems by partitioning on all dimensions and then applying a merge procedure. As our experiments show, the cost for the additional merge is more than compensated for by better overall performance and scalability.

To partition data, we use parallel sample sort [14]. As discussed in [14], one global data movement via one single  $h$ -relation is often sufficient to obtain sorted and well balanced data. The subsequent “global shift” operation, which needs another  $h$ -relation, is not always necessary. In our implementation of parallel sample sort we measure the imbalance of the sizes of the local data sets after the first  $h$ -relation and perform a second “global shift”  $h$ -relation only if necessary. Let  $y_0, \dots, y_{p-1}$  be the sizes of the sets  $Y_0, \dots, Y_{p-1}$  created on processors  $P_0, \dots, P_{p-1}$ , respectively, after the first  $h$ -relation. We calculate the relative imbalance  $\mathcal{I}(y_0, \dots, y_{p-1}) = \max\{(y_{max} - y_{avg})/y_{avg}, (y_{avg} - y_{min})/y_{avg}\}$ , where  $y_{max}$ ,  $y_{min}$ , and  $y_{avg}$  are the maximum, minimum and average of  $y_0, \dots, y_{p-1}$ , respectively. If  $\mathcal{I}(y_0, \dots, y_{p-1}) > \gamma$  for some threshold value  $\gamma$ , we apply a subsequent “global shift” operation. In our implementation we use a threshold value of  $\gamma = 1\%$ . As discussed in [14], the imbalance after the first  $h$ -relation is less if there are no duplicate keys. However, in most data, there are many duplicate values. Therefore, in Step 1a of Procedure 1, we first compute locally on each processor  $P_j$  ( $j = 0, \dots, p-1$ ) the  $D_i$ -root for its subset of data. This eliminates all duplicate keys  $D_i \dots D_{d-1}$  for the sort in the subsequent Step 1b.

We refer to our sample sort implementation as Adaptive-Sample-Sort. Since there are so many “folk” versions of parallel sample sort in the literature, we briefly review the exact sequence of steps implemented in our system.

**Procedure 2** Adaptive-Sample-Sort( $X_0, \dots, X_{p-1}; D_{i_1}, \dots, D_{i_k}; \gamma$ )

**Input:** Sets  $X_0, \dots, X_{p-1}$  stored on processors  $P_0, \dots, P_{p-1}$ , respectively.

**Output:** Sets  $X_0, \dots, X_{p-1}$  globally sorted by dimensions  $D_{i_1}, \dots, D_{i_k}$ .

- (1) Each processor  $P_j$  ( $j = 0, \dots, p-1$ ) locally sorts  $X_j$  by  $D_{i_1}, \dots, D_{i_k}$  and selects a set of  $p$  local pivots consisting of the elements with rank 0,  $(n/p^2), \dots, ((p-1)n/p^2)$ . Each processor  $P_j$  then sends its local pivots to processor  $P_0$ .
- (2) Processor  $P_0$  sorts the  $p^2$  local pivots received in the previous step. Processor  $P_0$  then selects a set of  $p-1$  global pivots consisting of the elements with rank  $(p + \lfloor p/2 \rfloor), (2p + \lfloor p/2 \rfloor) \dots ((p-1)p + \lfloor p/2 \rfloor)$  and broadcasts the  $p$  global pivots to all other processors.
- (3) Using the  $p-1$  global pivots received in the previous step, each processor  $P_j$  ( $j = 0, \dots, p-1$ ) locally partitions  $X_j$  (sorted by  $D_{i_1}, \dots, D_{i_k}$  from Step 1) into  $p-1$  subsequences  $X_j^0 \dots X_j^{p-1}$ .
- (4) Using one global  $h$ -relation, every processor  $P_j$ ,  $j = 0 \dots p-1$ , sends each  $X_j^k$ ,  $k = 0 \dots p-1$ , to processor  $P_k$ .
- (5) Each processor  $P_j$ ,  $j = 0, \dots, p-1$ , receiving  $p$  sorted sequences  $X_k^j$ ,  $k = 0 \dots p-1$ , in the previous step, locally merges those sequences into a single sorted sequence  $Y_j$  and sends the size  $y_j$  of  $Y_j$  to processor  $P_0$ .
- (6) IF  $\mathcal{I}(y_0, \dots, y_{p-1}) > \gamma$ , as determined by processor  $P_0$  THEN all processors  $P_0, \dots, P_{p-1}$  balance the sizes of  $Y_0, \dots, Y_{p-1}$  via a “global shift”, implemented by one  $h$ -relation operation.

Following the above global sort, each processor  $P_j$  ( $j = 0, \dots, p-1$ ) applies in Step 1c of Procedure 1 a sequential scan to its data set in order to compute the  $D_i$ -root ( $D_i$ -root $||_j$ ) for its local data.

## 2.3 Computation Of Local $D_i$ -Partitions

In this section, we discuss Step 2 of Procedure 1. The goal of this step is to compute on each processor  $P_j$  the  $D_i$ -partition with respect to  $D_i$ -root $||_j$ . For this, we apply on each processor a sequential top-down data cube construction method. Such methods, like Pipesort, typically consist of two phases. In the first phase, a schedule tree  $T_i$  is constructed. The nodes of  $T_i$  are the view identifiers of the  $D_i$ -partition and an edge  $(u, v)$  from parent  $u$  to child  $v$  indicates that  $v$  is created from  $u$ . Each edge  $(u, v)$  is labelled

“scan” or “sort”. If  $v$  is a prefix of  $u$ , then  $v$  can be created via a linear scan of  $u$ . If  $v$  is only a subset of  $u$  (but not a prefix), then the computation of  $v$  requires a re-sort of  $u$ . Sequential top-down cube construction methods like Pipesort attempt to build a schedule tree  $T_i$  that minimizes the work required for cube construction.

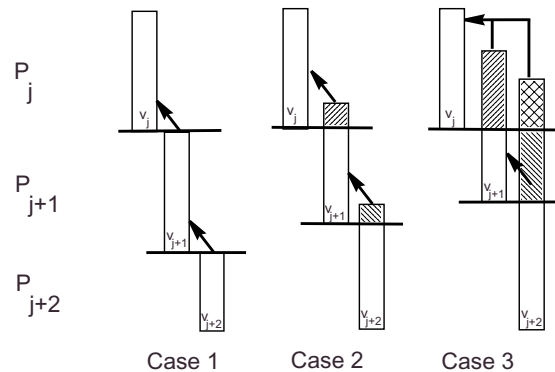
For shared-nothing parallel data cube construction, a problem that arises is that each processor  $P_j$  has a different data set, namely  $D_i\text{-root}|_j$ , and that the schedule trees can be different for these different sets. Indeed, the computation of the schedule tree is usually very much data driven. Pipesort and most other methods make statistical estimates of the view sizes, based on the data available, and schedule tree construction is based on those view sizes. In our case, we could allow each processor  $P_j$  to build its own local schedule tree for its local data set  $D_i\text{-root}|_j$  and build its  $D_i$ -partition accordingly. However, different local schedule trees for different processors imply that views of the  $D_i$ -partition created on different processors may be in different sort orders. This creates a problem during the subsequent merge phase in Step 3 of Procedure 1. When views of the same partition but for different subsets of data (i.e. on different processors) need to be merged, they need to have the same sort order or one of them has to be re-sorted. That re-sort creates a large amount of additional computation. Another possibility is to let one processor, say  $P_0$ , build the schedule tree for its data set  $D_i\text{-root}|_0$ , broadcast that schedule tree, referred to as the *global schedule tree*, and then let all processors use the same global schedule tree for their local cube construction. The advantage of this method is that we do not need to change the sort order of views during the merge. A potential disadvantage is that the sequential, local, top-down data cube methods (e.g. Pipesort) may not be using the “optimal” schedule tree for their data set. Recall that, the schedule trees generated by Pipesort and other top-down sequential methods are based on size estimates. As discussed in Section 4.2, our experiments indicate that, among the above two approaches, the latter method is far superior. For the data sets that we tested, the additional work on some processors because of non-optimal global schedule trees was much less than the overhead created through the need to re-sort views during the merge in Step 3. Therefore, Steps 2a, 2b and 2c of Procedure 1 implement the latter *global schedule tree* method.

## 2.4 Merge Of Local $D_i$ -Partitions

At the end of Step 2 of Procedure 1, each processor  $P_j$  has computed the  $D_i$ -partition for its local data set. For a view  $v$  of the  $D_i$ -partition, let  $v_j$  be the view created by processor  $P_j$ . In Step 3 of Procedure 1 we need to merge, for each view  $v$  in the  $D_i$ -partition, the  $p$  different views  $v_j$  created on the  $p$  different processors  $P_j$ . This merge is performed in Procedure *Merge-Partitions*( $D_i$ ) which will be discussed in the remainder of this section.

Consider Procedure 1 for  $i = 0$  and the A-partition shown in Figure 3. In Step 1 of Procedure 1, the A-roots are globally sorted by ABCD. Then, in Step 2, each processor  $P_j$  computes locally the A-partition for its data set. Consider the views  $ABCD_j$ ,  $ABC_j$ ,  $AB_j$ , and  $A_j$  computed in Step 2. All these views are in the same sort order as the global sort order created in Step 1 because they are a prefix of ABCD. We shall refer to these views as the *prefix views*. The other views,  $ABD_j$ ,  $AC_j$ ,  $ACD_j$  and  $AD_j$ , are not a prefix of ABCD and are therefore in a sort order that is different from the global sort order. We shall refer to them as the *non-prefix views*.

Consider a prefix view  $v$  and the problem of merging  $v_0, \dots, v_{p-1}$  stored on processors  $P_0, \dots, P_{p-1}$ . For example, consider the view  $v = AB$  in Figure 3 and the problem of merging  $AB_0, \dots, AB_{p-1}$ . The goal is to obtain a global AB sort order for  $AB_0 \cup AB_1 \dots \cup AB_{p-1}$  and then agglomerate those items that have the same values for dimensions A and B. Since AB is a prefix of the global sort order, ABCD, the first part is already done and the only items that, potentially, need to be agglomerated are the last item of  $v_j$  and the first item of  $v_{j+1}$  for each  $0 \leq j < p - 1$ . Therefore, in Procedure *Merge-Partitions*( $D_i$ ), for each prefix view  $v$  every processor  $P_{j+1}$  simply sends the first item of  $v_{j+1}$  to processor  $P_j$  which compares it with the last item of  $v_j$ . Nothing else needs to be done in order to merge all  $v_j$ . Figure 4 illustrates the case of a prefix view  $v$  as “Case 1”.



**Figure 4. Illustration of cases in Procedure Merge-Partitions**

We now study the case of merging the views  $v_0, \dots, v_{p-1}$  stored on processors  $P_0, \dots, P_{p-1}$  for a non-prefix view  $v$ . For example, consider the view  $v = AC$  in Figure 3 and the problem of merging  $AC_0, \dots, AC_{p-1}$ . Again, the goal is to obtain a global AC sort order for  $AC_0 \cup AC_1 \dots \cup AC_{p-1}$  and then agglomerate those items that have the same values for dimensions A and C. However, AC is *not* a prefix of ABCD and, therefore, the different  $v_j$  can have considerable overlap with respect to AC order. Figure 4 illustrates

the case of a non-prefix view  $v$  as “Case 2” and “Case 3”. The rectangles represent the  $v_j$  with respect to AC order. The shaded areas represent the overlap which, in contrast to Case 1 (prefix view), can now be considerably more than just one element. In Procedure Merge-Partitions( $D_i$ ), for each non-prefix view  $v$  every processor  $P_j$  sends its last element to every other other processor. Each processor  $P_k$  then determines its overlap with each  $P_j$  and sends that overlap to  $P_j$ . For each processor  $P_j$  let  $v'_j$  be the view  $v_j$  plus all the overlap received by processor  $P_j$ . We distinguish two cases which are both illustrated in Figure 4. “Case 2”: IF  $\mathcal{I}(|v'_0|, |v'_1|, \dots, |v'_{p-1}|) \leq \gamma$  for a non-prefix view  $v$  THEN each  $P_j$  locally sorts  $v'_j$  and agglomerates the items with same values for dimensions in  $v$ . “Case 3”: IF  $\mathcal{I}(|v'_0|, |v'_0|, \dots, |v'_0|) > \gamma$  for a non-prefix view  $v$  THEN the  $v_j$  are merged by a global sort. The distinguishing criterion between Cases 2 and 3 is the balance between the  $v'_j$ . If the imbalance is smaller than  $\gamma$  (Case 2) then we proceed similar to Case 1. If the imbalance is larger than  $\gamma$  (Case 3) then we need to completely re-balance via a global sort. In fact, for Case 3 we do not wish to even route the overlap between processors. We rather re-sort immediately. Hence, in order to determine whether Case 2 or Case 3 applies, each processor  $P_k$  first determines the *size* of its overlap with each  $P_j$  and sends only the information about the size of that overlap to  $P_j$ .

The following is an outline of Procedure Merge-Partitions( $D_i$ ).

#### Procedure 3 Merge-Partitions( $D_i$ )

- (1) For each view  $v \in DC_i$ , each processor  $P_j$  broadcasts the last item of  $v_j$  to every other processor  $P_k$  and receives back the sizes of all overlaps.
- (2) For each view  $v \in DC_i$ , every processor  $P_j$  determines  $|v'_j|$  and sends all of its  $|v'_j|$  values to processor  $P_0$ .
- (3) Processor  $P_0$  determines for each view  $v \in DC_i$  whether it is a “Case 1”, “Case 2”, or “Case 3”.
- (4) Every processor  $P_{j+1}$  sends for each “Case 1” view  $v \in DC_i$  the first item of  $v_{j+1}$  to processor  $P_j$ , and  $P_j$  compares/agglomerates that item with the last item of  $v_j$ .
- (4) Every processor  $P_k$  sends for each “Case 2” view  $v \in DC_i$  its overlap with every  $v_j$  to the respective processor  $P_j$ . Every processor  $P_j$  merges/agglomerates all received overlaps with  $v_j$ .
- (5) All remaining “Case 3” views  $v \in DC_i$  are merged via global sort, using Procedure 2 with  $\gamma = 3\%$ .

In the following, we discuss a small extension of Procedure Merge-Partitions( $D_i$ ). Note that, all data sets are stored in secondary memory. For Step 2, a straight forward implementation would imply that the entire disk needs to be scanned on each processor in order to determine the  $|v'_j|$  values. However, we do actually only require a  $1/p\%$  accuracy

for the  $|v'_j|$  values in order to obtain a 1 % accuracy for the imbalance  $\mathcal{I}$  which is sufficient for distinguishing between cases 2 and 3. Hence, it is sufficient to use a sample of only  $100p$  equal spaced sample elements of the locally sorted  $v_j$  instead of the entire  $v_j$ . Such a sample can be build during the local computation of  $v_j$  in Step 2c of Procedure 1. Note that, while  $P_j$  writes  $v_j$  to its local disk, the size of  $v_j$  is not yet known and hence, the size of the sample and the distance between sample elements is not yet known. This problem can be solved as follows. A sample array  $A[1..a]$  of size  $a = 100p$  is allocated in main memory. While the first  $a$  elements of  $v_j$  are written to disk, each of them is also copied into  $A$ . While the second  $a$  elements of  $v_j$  are written to disk, every second is written into every second location of  $A$ , overwriting the previous element stored at that location. While the third and fourth groups of  $a$  elements of  $v_j$  are written to disk, every fourth is written into every second location of  $A$ , and so on. When the entire view  $v_j$  is written to disk, an appropriate sample will be available in array  $A$  to determine the  $|v'_j|$  values with sufficient accuracy.

### 3 Partial Data Cube Construction On Shared-Nothing Multiprocessors

Our method has the advantage that it is easily extended to the case where not the entire data cube but only a subset of selected views are to be computed. This case occurs frequently in practice because the user often knows that some views will not be required for the subsequent OLAP queries that are executed on the data cube. For example, for a raw data set with 20 dimensions, it may be clear from the application that the OLAP queries will only require views with at most 5 dimensions. Therefore, it would be wasteful to create all  $2^{20}$  views when most of them are never used.

For the purpose of computing a partial data cube, we re-define  $S$ . Instead of being the set of all  $2^d$  view identifiers (as defined in Section 2.1), we define  $S$  to be the view identifiers of the subset of selected views. The definitions of  $S_i$ ,  $DC_i$ , etc. are then all with respect to the new set  $S$  of *selected* views. The algorithm in Section 2 remains completely unchanged except for the construction of the schedule tree  $T_i$  in Step 2a of Procedure 1. Instead of using the first phase of an arbitrary sequential top-down data cube method, we apply the sequential schedule tree construction method for partial cubes that we have recently presented in [4]. For a set  $S$  of selected views, our algorithm in [4] can create a schedule tree that is either a subtree of the tree that would be generated by Pipesort for the entire cube, or it can create a schedule tree directly from the lattice. An example of a schedule tree for a partial cube is shown in Figure 1c. Note that, for optimal performance, some “intermediate” views need to be constructed in addition to the selected views.

## 4 Performance Evaluation

We have implemented our parallel shared-nothing data cube generation method using C++ and the MPI communication library. This implementation evolved from the code base for a fast sequential Pipesort [3] and the sequential Partial cube method described in [4]. Most of the required sequential graph algorithms, as well as data structures like hash tables and graph representations, were drawn from the LEDA library [13].

Our experimental platform consisted of a 16 node Beowulf cluster with 1.8 GHz Intel Xeon processors, 512 MB RAM per node and two 40 GB 7200 RPM IDE disk drives per node. Every node was running Linux Redhat 7.2 with gcc 2.95.3 and MPI/LAM 6.5.6. as part of a ROCKS cluster distribution. All nodes were interconnected via an Intel 100 Megabyte Ethernet switch. Note that on this machine communication speed is extremely slow in comparison to computation speed. We will shortly be replacing our 100 Megabyte interconnect with a 1 Gigabyte Ethernet interconnect and expect that this will further improve the relative speedup results obtainable on this machine.

In the following experiments all sequential times were measured as wall clock times in seconds. All parallel times were measured as the wall clock time between the start of the first process and the termination of the last process. We will refer to the latter as *parallel wall clock time*. All times include the time taken to read the input from files and write the output into files. Furthermore, all wall clock times were measured with no other user except us on the Beowulf cluster.

In our experimentation we generated a large number of synthetic data sets which varied in terms of the following parameters:  $n$  - number of records,  $d$  - number of dimensions,  $|D_0|, |D_1| \dots |D_{d-1}|$  - cardinality in each dimension, and  $\alpha_0, \alpha_1 \dots \alpha_{d-1}$  - skew in each dimension.

Our experiments explored the following six performance issues:

1. **Relative Speedup:** We investigated the effect of increasing the number of processors on the time required to solve data cube generation problems and measured the relative speedup, i.e. the ratio between observed sequential time and observed parallel time. Sequential times for computing full cubes and partial cubes were measured on a single processor of our parallel machine using our sequential implementations of Pipesort [3] and Partial cube [4], respectively.
2. **Local vs. global schedule trees:** We compared the effect on parallel wall clock time of using local vs. global schedule trees.
3. **Data skew:** We investigated the effect on parallel wall clock time of data sets with varying skewed distributions. We used the standard ZIPF [26] distribution with  $\alpha = 0$  (no skew) to  $\alpha = 3$  (high skew) and explored the relationship between data skew and the amount of data that must be communicated.
4. **Cardinality of dimensions:** We investigated the effect of varying dimension cardinalities on parallel wall clock time for both skewed and non-skewed data sets.
5. **Data dimensionality:** We investigated the effect of varying dimensionality, and therefore the effects of relative density or sparsity, on parallel wall clock time.
6. **Balance Tradeoffs:** Lastly, we investigated the effect of varying the balance threshold parameter  $\gamma$ . As  $\gamma$  is decreased we improve the balance in the distribution of views across processors, but at the cost of more data movement.

### 4.1 Relative Speedup

Speedup experiments are at the heart of the experimental evaluation of our parallel shared-nothing data cube generation method. They consist of incrementally increasing the number of processors available to our data cube generation software to determine the parallel speedup obtained. Figure 5 shows for *full cube construction* the parallel wall clock time observed for data sets of varying sizes as a function of the number of processors used, and the corresponding relative speedup. We observe that for an input size  $n = 2,000,000$  rows (72 Megabytes) our method achieves close to optimal speedup; generating a full data cube of  $\approx 227$  million rows (5.6 Gigabytes) in just under 6 minutes on 16 processors. The speedup for smaller problems is lower as there is insufficient local computation over which to amortize the cost of communications. On the other hand, our method works well on very large data sets. For example, on an input data set of 10,000,000 rows (360 Megabytes), our parallel method created the corresponding data cube consisting of  $\approx 846$  million rows (21.7 Gigabytes), on 16 processors, in under 47 minutes.

Note that our speedup results could be further improved by overlapping communication and local computation. Our current implementation does not overlap the local computation of  $D_i$ -Partitions with the global communication involved in merging  $D_{i-1}$ -Partitions. Doing so would mask between 40% and 60% of the communication overhead and further improve the speedup results.

Figure 6 shows for *partial cube construction* the parallel wall clock time observed for a range of different percentages of selected views as a function of the number of processors, and the corresponding relative speedup. We observe that when 50% or more of the views are selected, the speedup obtained decreases somewhat in comparison to the full cube case. However, for as little as 25% selected views, the speedup obtained is still more than half of optimal. Only when the number of views selected gets very



small, approaching  $d$ , speedup falls off rapidly as the local work within partitions is little more than the computation of the root view. In such cases, when there are only a handful of selected views, creating each view from an independent sort of the original data set may be preferable.

## 4.2 Local vs. global schedule trees

As described in Section 2.3, for shared-nothing parallel data cube construction it is possible for each processor to use either a local or a global schedule tree. Local schedule trees are built by each processor  $P_j$  relative to their own data set  $D_i\text{-root}|_j$ , whereas a global schedule tree is built by a single processor, say  $P_0$ , relative to its data set  $D_i\text{-root}|_0$ , and then broadcast to all other processors.

The use of local schedule trees might appear at first preferable, since they are optimized relative to a processor's own data set. However, they have one serious drawback. When views of the same partition but for different subsets of data (i.e. on different processors) need to be merged, they need to have the same sort order or one of them has to be re-sorted. That re-sort creates a large amount of additional computation. As can be seen in Figure 7, our experiments indicate that local schedule trees offer superior performance in practice. For the data sets that we tested, the additional work on some processors because of non-optimal schedule trees was significantly less than the overhead created through the need to re-sort views during the Merge-Partitions() procedure.

## 4.3 Data skew

Data sets with skewed distributions can pose an interesting challenge to parallel data cube generation methods. As skew increases, data reduction tends also to increase, particularly in top-down generation methods [20, 1]. Data reduction is typically positive, as it reduces the total amount of work to be performed. However, if data reduction is large and unevenly spread across the processors it may unbalance the computation and cause the amount of data that has to be communicated to rise sharply.

To explore this issue we generated data sets using the standard ZIPF [26] distribution in each dimension with  $\alpha = 0$  (no skew) to  $\alpha = 3$  (high skew). Figure 8a shows the impact of skew on parallel wall clock time. Figure 8b shows, for the same data sets, how skew affects the overall size of the data that must be communicated in performing the Merge-Partitions() procedure. We observe that, in general, as skew is increased parallel time decreases due to data reduction and decreased local computation. For  $\alpha = 1$  there is a sharp rise in the amount of data to be communicated, which offsets some gains from the reduced local computation. However for  $\alpha > 1$  the data reduction is so significant that only very little data needs to be communicated and parallel time drops significantly.

## 4.4 Cardinality of dimensions

The cardinality of the dimensions in a data set can affect the performance of our method. As cardinalities increase so does the sparsity of the data set and this may adversely effect parallel time especially given that top-down methods [20, 1] are designed primarily for dense data cubes. Curves A, B and C of Figure 9a clearly illustrate this effect. The sparser data sets require somewhat more time, although, as can be seen in Figure 9b, this has little effect on the relative speedup achieved.

A close examination of the details of our algorithm suggests that a potentially difficult input data set for our method would be one in which the leading dimension has high skew and large cardinality, while the remaining dimensions have low skew. In such cases, the global sort used to create the  $D_0\text{-root}$  may do little to reduce the amount of communication required in building the views in  $DC_0$ . Curve D in Figure 9 shows the results measured for such a case. We observe that whereas such situations do reduce the speedup obtained, the reduction is relatively small. Even for this difficult data set, the speedup obtained by our method is still close to half of the optimal speedup.

## 4.5 Data dimensionality

Figure 10 shows parallel wall clock time in seconds as a function of the dimensionality of the raw data set. Note that, the number of views that must be computed grows exponentially with respect to the dimensionality of the data set. In Figure 10, we observe that the parallel running time grows essentially linearly with respect to the output size.

## 4.6 Balance tradeoffs

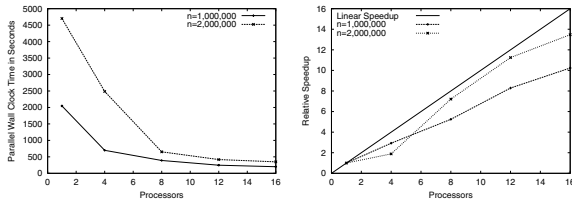
One important feature of our shared-nothing data cube generation algorithm is that it balances *each* view in the generated data cube over the processors within a balance threshold  $\gamma$ . The more balanced each view is across the processors (i.e. the smaller  $\gamma$ ) the more balanced any subsequent parallel computation on each view will be. However, the cost of selecting a small  $\gamma$  is that it may cause more data movement and therefore increase the time required for data cube generation.

Figure 11 shows parallel wall clock time in seconds as a function of the number of processors for a range of different balance thresholds, as well as the corresponding speedup curves. We observe that while reducing the balance threshold  $\gamma$  increases the parallel time, the effect is small. A  $\gamma$  of 3% appears to be a good threshold in practice. However, individual applications may want to tune this parameter according to their needs and the performance characteristics of their parallel machines.

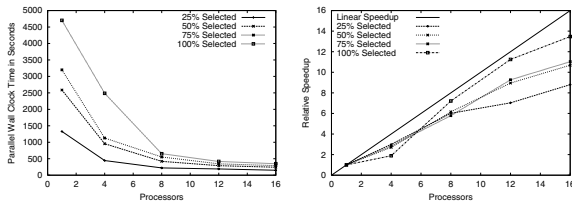


## 5 Conclusion

In this paper, we study parallel data partitioning methods for ROLAP data cubes that can be executed on shared-nothing multiprocessors. The principal advantage of ROLAP is that it allows for tight integration with current relational database technology. We have implemented our parallel data cube method and evaluated it on a PC cluster, exploring relative speedup, local vs. global schedule trees, data skew, cardinality of dimensions, data dimensionality, and balance tradeoffs. We obtained promising speedup results for a wide range of input data sets. We are currently exploring the integration of our method with commercial parallel database systems.



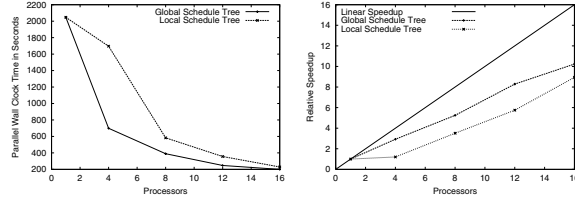
**Figure 5.** (a) Parallel wall clock time in seconds as a function of the number of processors for data of size  $n = 1,000,000$  rows and  $n = 2,000,000$  rows and (b) corresponding speedup. (Fixed parameters: Dimensions  $d = 8$ . Cardinalities  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Skew  $\alpha = 0$ . Percentage of views selected  $k = 100\%$ .)



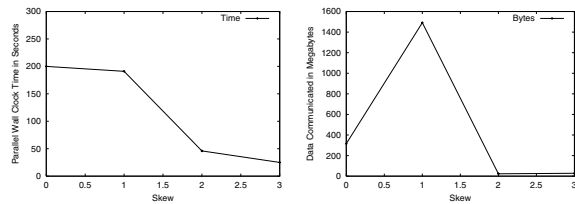
**Figure 6.** (a) Parallel wall clock time in seconds as a function of the number of processors for a range of different percentages of selected views and (b) corresponding speedup. (Fixed parameters: Data size  $n = 2,000,000$  rows. Dimensions  $d = 8$ . Cardinalities  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Skew  $\alpha = 0$ .)

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**Figure 7.** (a) Parallel wall clock time in seconds as a function of the number of processors for local and global schedule tree methods and (b) corresponding speedup. (Fixed parameters: Data size  $n = 1,000,000$  rows. Dimensions  $d = 8$ . Cardinalities  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Skew  $\alpha = 0$ . Percentage of views selected  $k = 100\%$ .)



**Figure 8.** (a) Parallel wall clock time in seconds as a function of the skew for  $\alpha = 0, 1, 2, 3$ , and (b) and the size of corresponding data movements. (Fixed parameters: Data size  $n = 1,000,000$  rows. Dimensions  $d = 8$ . Cardinalities  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Percentage of views selected  $k = 100\%$ . Number of processors  $p = 16$ .)

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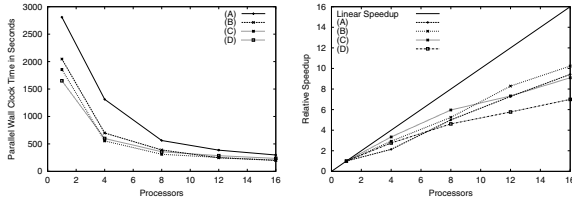
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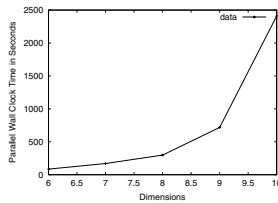
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**Figure 9.** (a) Parallel wall clock time in seconds as a function of the number of processors for data sets with different cardinality mixes, and (b) corresponding relative speedup. (Fixed parameters: Data size  $n = 1,000,000$  rows. Dimensions  $d = 8$ . Cardinalities and skews (A)  $|D_i| = 256, 256, 256, 256, 256, 256, 256, 256$ . Skew  $\alpha = 0$ . (B)  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Skew  $\alpha = 0$ . (C)  $|D_i| = 16, 16, 16, 16, 16, 16, 16, 16$ . Skew  $\alpha = 0$ . (D)  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Skew  $\alpha_0 = 3$  and  $\alpha_{i>0} = 0$ .)



**Figure 10.** Parallel wall clock time in seconds as a function of the the number of dimensions. (Fixed parameters: Data size  $n = 1,000,000$  rows. Cardinalities  $|D_i| = 256$  in all dimensions. Percentage of views selected  $k = 100\%$ . Number of processors  $p = 16$ .)

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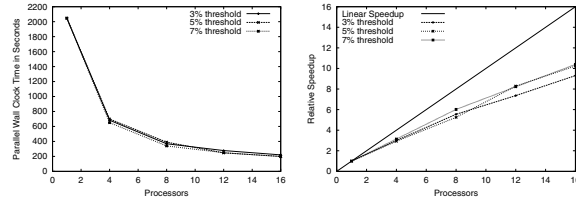
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**Figure 11.** (a) Parallel wall clock time in seconds as a function of the number of processors for a range of different balance thresholds  $\gamma$  and (b) corresponding speedup. (Fixed parameters: Data size  $n = 1,000,000$  rows. Dimensions  $d = 8$ . Cardinalities  $|D_i| = 256, 128, 64, 32, 16, 8, 6, 6$ . Skew  $\alpha = 0$ . Balance threshold  $\gamma = 3\%, 5\%$ , and  $7\%$ .)

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