Approximating Weighted Shortest Paths on Polyhedral Surfaces*

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1 Introduction

Shortest path problems are among the fundamental problems studied in computational geometry. In this video, we consider the problem of computing a shortest cost path between two points s and t on a (possibly non-convex) polyhedral surface $\mathcal{P}$. The surface is composed of triangular regions (faces) in which each region has an associated positive weight. The cost of travel through each region is the distance traveled times its weight.

The computation of Euclidean shortest paths on non-convex polyhedra has been investigated by [5, 2, 1]; currently, the best known algorithm due to Chen and Han [2] runs in $\mathcal{O}(n^2)$ time. Mitchell and Papadimitriou [6] introduced the Weighted Region Problem and an algorithm that computes a shortest weighted cost path between two points in a planar subdivision; it requires $\mathcal{O}(n^8 \log n)$ time in the worst case. They state that their algorithm applies to non-convex polyhedral terrains with modifications.

Most shortest path applications demand simple and efficient algorithms to compute approximate shortest paths as opposed to a complex algorithm that computes an exact path. Polyhedra arising in these applications approximate real surfaces and thus an approximate path will typically suffice. Our interest is also motivated by our research and development on a parallel system for GIS and spatial modeling. For a more detailed explanation of the contents of this video, see [3] or [4].

[1] Research is supported by ALMERCO Inc. and the Natural Sciences and Engineering Research Council of Canada.

2 Video Contents

The video describes several schemes that allow the computation of approximate shortest paths on polyhedral surfaces in both the weighted and unweighted scenarios. The schemes are based on adding Steiner points along the polyhedral edges and interconnecting them across each face thereby building weighted graphs. The schemes vary in the way in which Steiner points and edges are added. An approximate shortest path for the polyhedral surface is obtained by applying Dijkstra's shortest path algorithm on the weighted graph. As more and more Steiner points are added, the path accuracy increases (bounds on the accuracy have been established in the accompanying paper). We show that with a constant number (around six) of Steiner points per polyhedral edge, the path cost converges to a near-optimal value.

We have implemented these schemes as well as Chen and Han's [2] shortest path algorithm. Each of the schemes is described in the video and we also give experimental results that show how well they perform on a test suite of terrain data. In the unweighted case, we compare the path accuracy and algorithm running time with our implementation of Chen and Han's algorithm. We present graphs showing that our schemes run (much) faster and obtain near-optimal path accuracy. Since we do not have an implementation of an algorithm to compute exact weighted shortest paths, we are unable to make a comparison of our weighted paths. We do show however, that our weighted paths exhibit similar convergence behaviour as in the unweighted case. We also give results showing the effects on the approximated paths when the terrain becomes more "spiky".

3 Experimental Testing

Our testing was performed on a special class of non-convex polyhedra called a *Triangular Irregular Network* (TIN). This is a typical terrain model used in Geographical Information Systems (GIS) to represent a 2.5-d surface in which each vertex is given a height value. One of the main difficulties in presenting experimental results is to choose *benchmark* TINs. It is conceivable that different TIN characteristics could affect the performance of an algorithm. We have attempted to accommodate different characteristics by performing our tests with TINs that have different sizes (i.e., 1012, 5000 and 10,082 faces), height characteristics (i.e., smooth or spiky (modeled by accentuating the heights)), and data sources (i.e., random or sampled from Digital Elevation Models (DEM)).

For each TIN, we computed a set of 100 randomly selected pairs of vertices. We then tested each of the approximation schemes. The tests were performed in iterations based on the number of Steiner points per edge. In each iteration, we computed a graph and then we computed the path cost between each of the 100 vertex pairs and then obtained an "average path cost" and "computation time" for these pairs. Each scheme was tested for both weighted and non-weighted scenarios.

4 Video Preparation

The video was created using Microsoft Powerpoint '97. The video contains some terrain images which were obtained from snapshots of our implementation which used XGL and Motif on a SUN Sparc 20. The authors thank Carleton University's Instructional Media Center for technical support.

References

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