COMP 2804 — Assignment 1

Due: Monday February 8, before 4:30pm, in the course drop box in Herzberg 3115. Note that 3115 is open from 8:30am until 4:30pm.

Assignment Policy: Late assignments will not be accepted. Students are encouraged to collaborate on assignments, but at the level of discussion only. When writing the solutions, they should do so in their own words. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams. Please show your work for each question.

Important note: When writing your solutions, you must follow the guidelines below.

- The answers should be concise, clear and neat.
- When presenting proofs, every step should be justified.
- Assignments should be stapled or placed in an unsealed envelope.
- All questions carry equal marks.

Substantial departures from the above guidelines will not be graded.

Question 1: On the first page of your assignment, write your name, student number, and course number COMP 2804.

Question 2: Consider a restricted alphabet set that is made of letters $\{A, C, T, G\}$. How many 5-letter words you can form from $\{A, C, T, G\}$ with the following restrictions (please justify your answer):

1. The word ends with A.
2. The word starts with A and ends with T.
3. The word contains only C and G.
4. The word does not contain C.
5. The word does not contain AAA in consecutive positions.
   (For example, CTAAA, TAAAC, AAAAC are not valid.)

Question 3: Let $n > 0$ be even. A binary string B is called a palindrome, if you obtain the same string, either you read the bits from left to right or from right to left. For example, the strings 001100 and 101101 are palindromes, whereas 010100 and 110001 are not palindromes. How many bit strings of length n are palindromes?

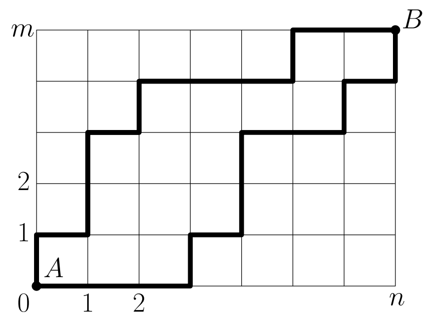
Question 4: A polygon on n-vertices is convex, if the segment joining any pair of points, either on the boundary or in the interior of the polygon, lies entirely within the polygon.

A diagonal in a polygon is a line segment connecting a pair of non-adjacent vertices. How many diagonals does a convex polygon on n vertices can have?

Question 5: A coin is flipped 8 times, and in each flip the outcome is either **H**ead or **T**ail. How many outcomes have:

1. Exactly four Heads?
2. At most four Heads?
3. Same number of Heads and Tails?
4. More Heads than Tails?

Question 6:



Let $n > 0$ and $m > 0$ be integers. Consider an $n \times m$ rectangle as shown in figure. We are interested in finding paths between the point $A = (0, 0)$ and $B = (n, m)$. A path starts at $A = (0, 0)$ and in each step, we are allowed to move one unit either in $+x$-direction or in $+y$-direction, till we reach $B = (n, m)$. Such type of paths are called monotone paths. (Examples of two such paths are shown in bold in the figure.) Note that any monotone path is composed of exactly n unit horizontal segments and m unit vertical segments. How many monotone paths are there from A to B? Justify your answer.

Question 7: We need to select a committee consisting of n members from a group n women and n men, such that the chairperson of the committee must be a woman. By counting in two different ways, the total number of ways such a committee can be formed, show that

$$n\binom{2n-1}{n-1} = \sum_{r=1}^{n} r\binom{n}{r}^2.$$

Question 8: Assume $x \neq 0$. What is the coefficient of x^1 and x^2 in the expansion of

$$\frac{(1+x)^n - 1}{x}$$

Question 9: Consider the following equation

$$(1+x)^{n+1} - 1 = x(1 + (1+x) + (1+x)^2 + \cdots + (1+x)^n)$$

1. Show that the equation holds for all positive values of n.
   For example you can prove this by using induction on n.

2. By looking at coefficients of the term x^2 in the equation, show that

$$\binom{n+1}{2} = \binom{1}{1} + \binom{2}{1} + \binom{3}{1} + \cdots + \binom{n}{1}.$$

3. Derive a similar type of expression by looking at the coefficients of the term x^3 in the equation, i.e., is it true that

$$\binom{n+1}{3} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \cdots + \binom{n}{2}.$$

Question 10: How many ways you can fill your bag with 12 fruits from the Caribbean fruit store which sells four types of fruits (Mango, Sugar-Apple, Jamun, and Guava)? You can assume that the store has more than 12 fruits of each type.