COMP 2804 — Assignment 3

Due: March 21, before 4:30pm, in the course drop box in Herzberg 3115. Note that 3115 is open from 8:30am until 4:30pm.

Assignment Policy: Late assignments will not be accepted. Students are encouraged to collaborate on assignments, but at the level of discussion only. When writing the solutions, they should do so in their own words. Past experience has shown conclusively that those who do not put adequate effort into the assignments do not learn the material and have a probability near 1 of doing poorly on the exams.

Important note: When writing your solutions, you must follow the guidelines below.

- The answers should be concise, clear and neat.
- When presenting proofs, every step should be justified.
- Assignments should be stapled or placed in an unsealed envelope.

Substantial departures from the above guidelines will not be graded.

Question 1: On the first page of your assignment, write your name and student number.

Question 2: You flip a fair coin four times. Define the four events (recall that zero is even)

$$\begin{aligned} A &= \text{"the coin comes up heads an odd number of times"}, \\ B &= \text{"the coin comes up heads an even number of times"}, \\ C &= \text{"the coin comes up tails an odd number of times"}, \\ D &= \text{"the coin comes up tails an even number of times"}. \end{aligned}$$

- Determine $\Pr(A)$, $\Pr(B)$, $\Pr(C)$, $\Pr(D)$, $\Pr(A \mid C)$, and $\Pr(A \mid D)$. Show your work.
- Are there any two events in the sequence A, B, C, and D that are independent? Justify your answer.

Question 3: Answer the following questions:

1. Suppose a fair coin is tossed 10 times. What is the probability of obtaining exactly 4 Heads in 10 tosses?
2. Suppose a fair die is rolled 10 times. What is the probability of getting "6" exactly 4 times?

Question 4: In Section 5.4.1, we have seen the different cards that are part of a standard deck of 52 cards.

- You get a uniformly random hand of three cards from a standard deck of 52 cards. Determine the probability that this hand contains an Ace, a King, and a Queen. Show your work.
- You get a uniformly random hand of three cards from the 13 spades. Determine the probability that this hand contains an Ace, a King, and a Queen. Show your work.

Question 5: Suppose in the Minor Hockey League Championship play off series between Nepean Pirates and Kanata Thunders, there are three possible playoff games planned. A team that wins two games is declared the champion. Outcome of each game is either a win or a loss - there are no ties! The first game is played in the Pirates Arena, the Second game is played in the Thunder's arena, and if required the third game will be played in the Pirates arena. A team wins with a probability of 2/3rd in its home arena, and with a probability of 1/3rd in the opposition's arena. What is the probability that Nepean Pirates will be declared the Champion?

Question 6: Suppose that 8% of all bicycle racers use steroids, that a bicyclists who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of times. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids?

Question 7: There are three cards in a box. Both sides of Card 1 are Black. Both sides of Card 2 are Red. One side of Card 3 is Black and the other side is Red. We pick a card uniformly at random and observe only one of its side. Answer the following questions:
(a) If the side we observed is Black, what is the probability that the other side is also black?
(b) What is the probability that the opposite side is the same color as the side that we observed?

Question 8: Suppose a store has seveal boxes of candies, where each box is either of Type A or Type B. The stores inventory consists of 60% of Type A boxes and 40% of Type B boxes. Each Type A box contains 70% sweet candies and 30% sour candies. Each Type B box contains 30% sweet candies and 70% sour candies. These boxes are not labelled, and hence by looking at a box we cannot determine which type it is. A customer goes to the store and decides to buy a box. The storekeeper hands a random box to this customer. The customer opens the box, tastes a random candy, and then needs to make a decision, whether the box is likely to be of Type A or Type B.

1. Determine the probability that if the candy the customer tasted is sweet, what is the probability that: It is a Type A box? It is a Type B box?
2. Determine the probability that if the candy the customer tasted is sour, what is the probability that: It is a Type A box? It is a Type B box.

Question 9: Three factories A, B, and C, produce 25%, 35%, and 40%, of the total output of snow shovels in a season, respectively. Out of their production, 5%, 4%, and 2%, respectively, are defective shovels. A shovel is chosen uniformly at random and found to be defective. What is the probability that this shovel came from: a) Factory A? b) Factory B?, c) Factory C?

Question 10: In a manufacturing process of an article, defects of one type occurs with probability 0.1 and the defects of other type occurs with probability 0.05. Assume that the two defects are independent of each other. What is the probability that

1. An article doesn't have both types of defects.
2. An article is defective.
3. An article has only one type of defect, given that it is defective.

Question 11: (Bonus Problem:) An airplane has seats for $n + k$ passengers, where $n \geq 2$ and $k \geq 0$ are integers. In all n passengers are going to board the flight, and each of them has been issued a boarding pass with their seat number. Passengers board the airplane one by one. The first passenger, when he boards the flight, observes that his seat number is not printed very well on his boarding pass, and hence occupies a random seat. The second passenger takes his seat, if it is empty, otherwise chooses one of the random unoccupied seats. The third passenger takes her seat, if it is empty, otherwise chooses one of the random unoccupied seats. And this process is repeated till all the n-passengers have boarded the flight. We can assume that the seat numbers are readable for all passengers, except for the first one. What is the probability that the last person finds his/her seat occupied?