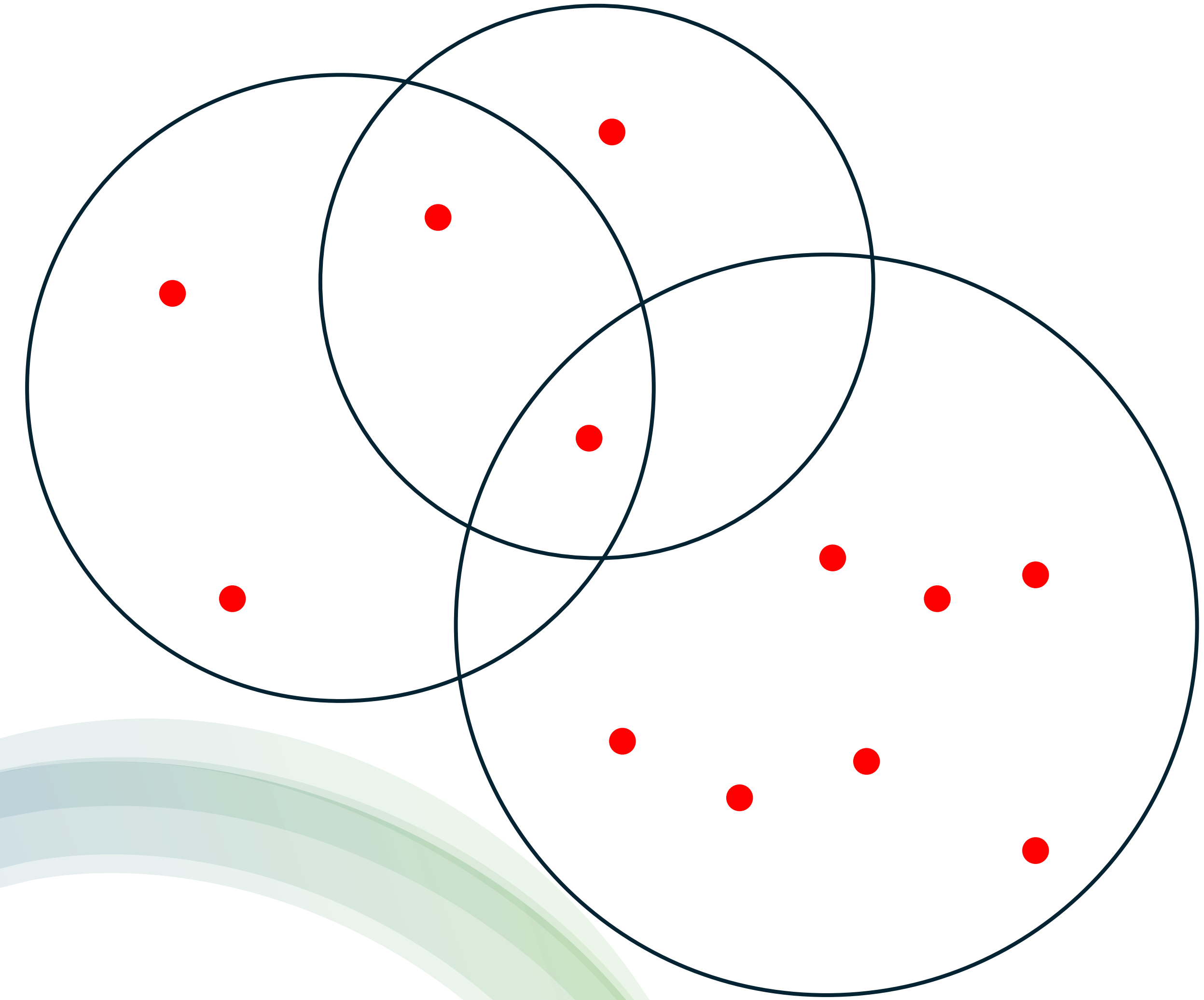
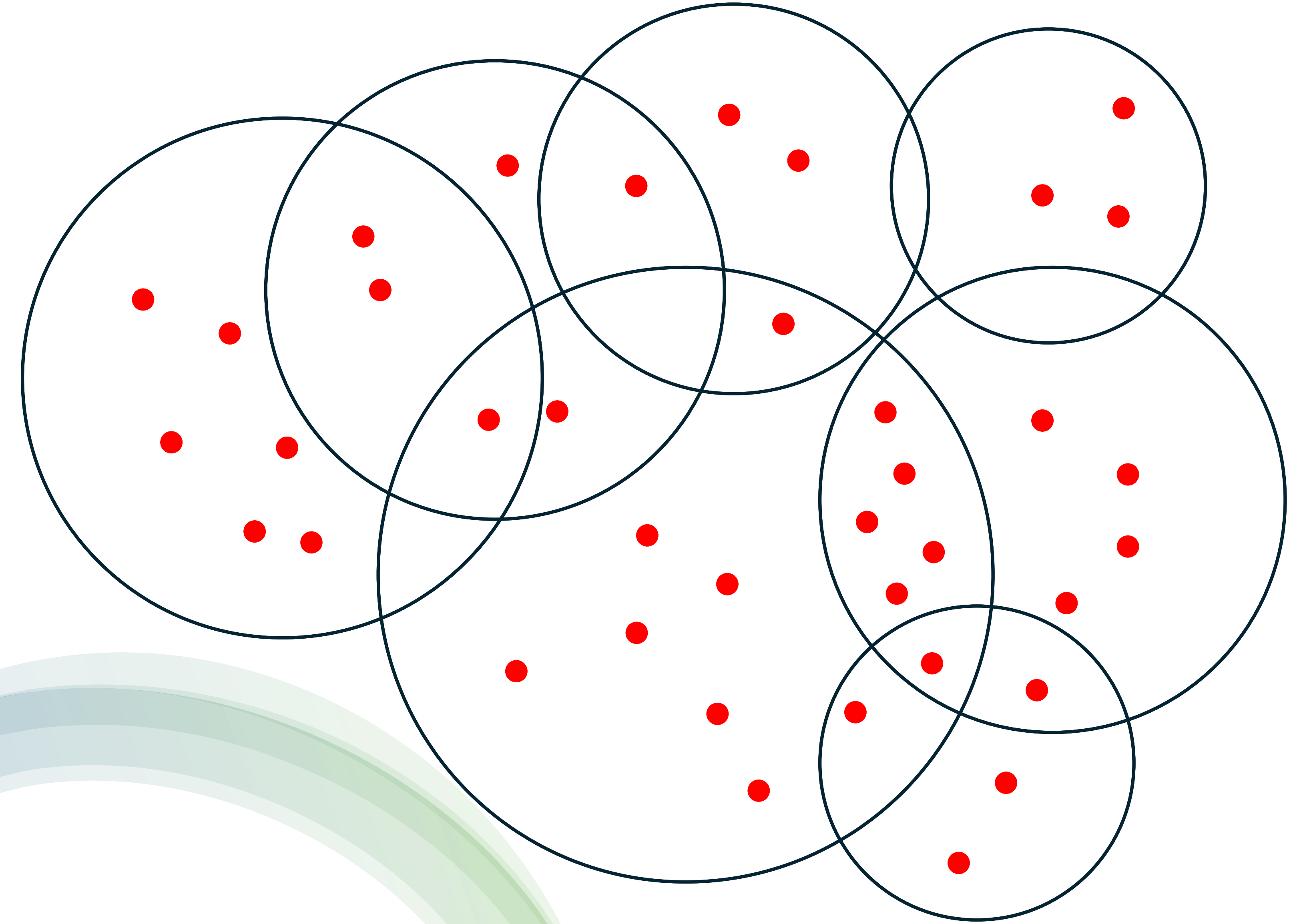
Maximum Coverage

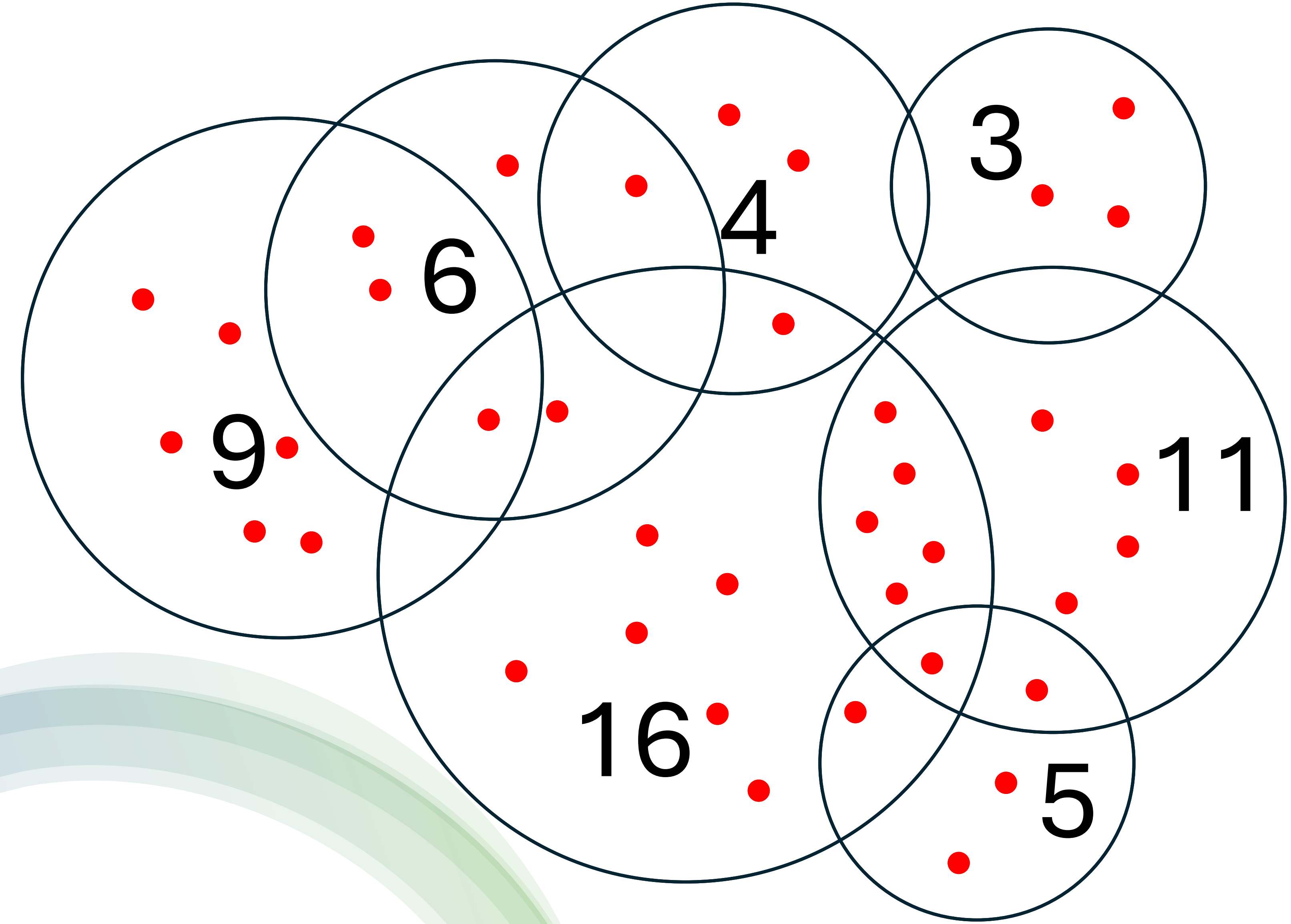
Chaplick, S., De, M., Ravsky, A., & Spoerhase, J.



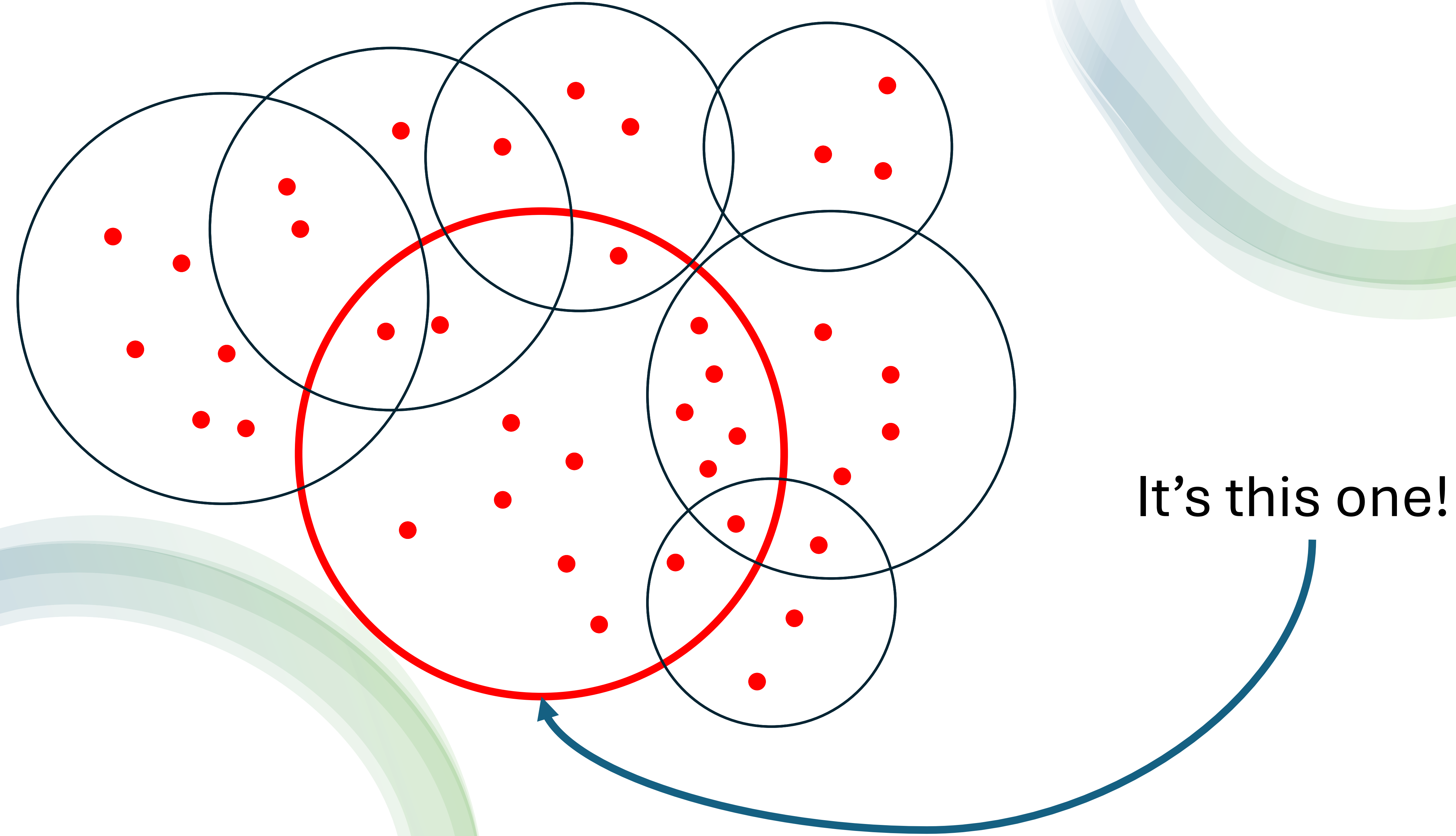
Select the k disks that cover the most points.

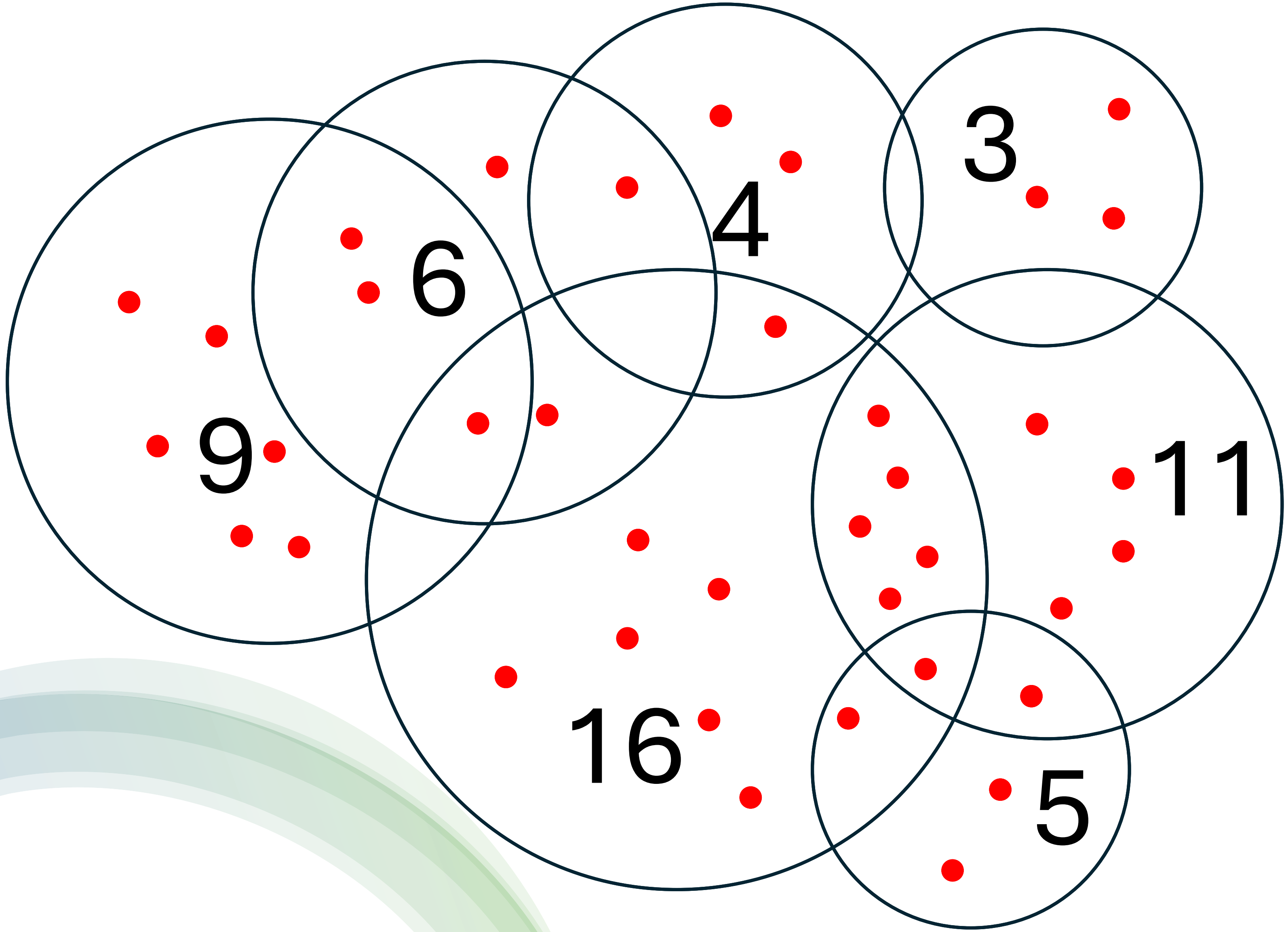


Select the 1 disk that covers the most points.

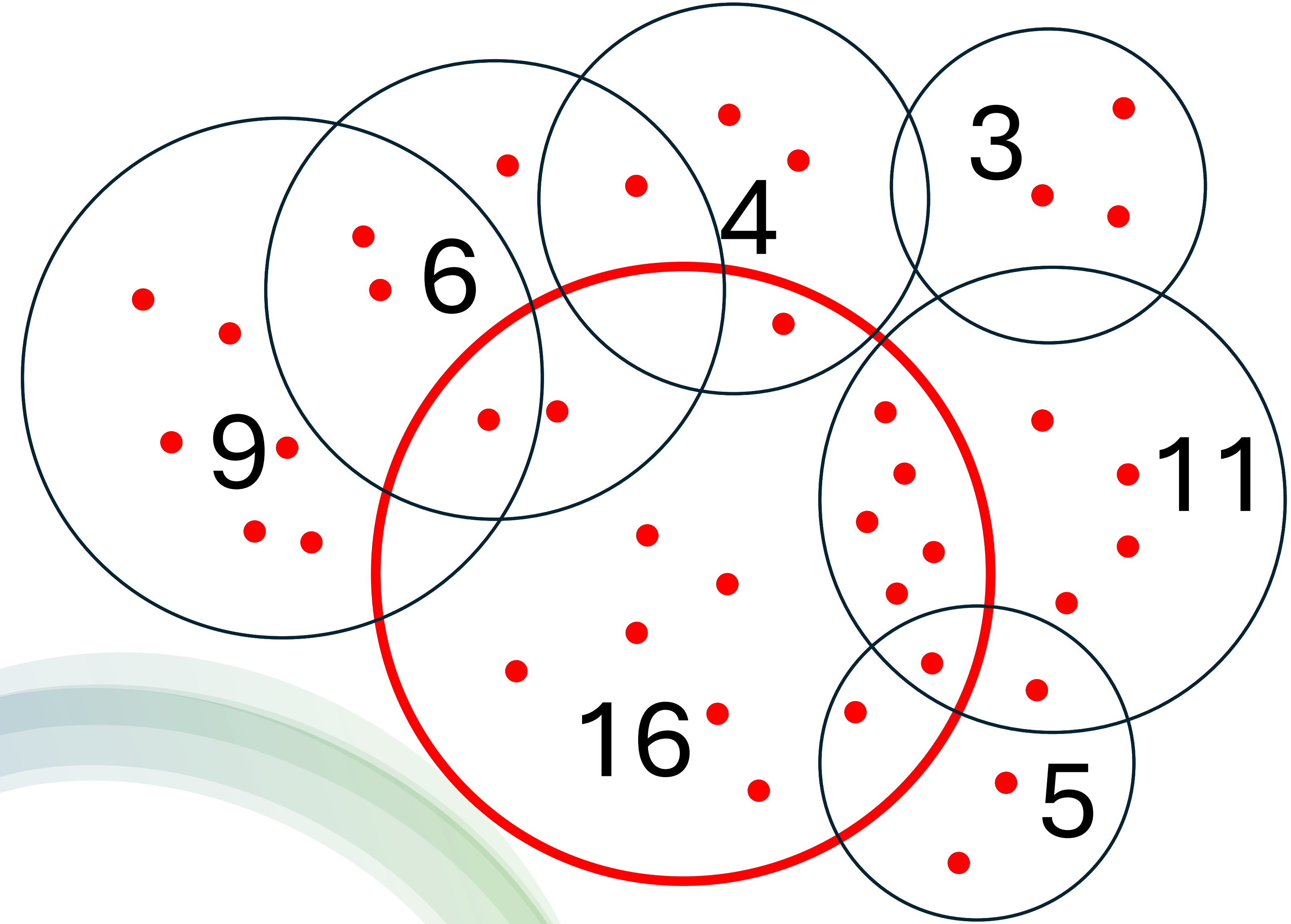


Select the 1 disk that covers the most points.

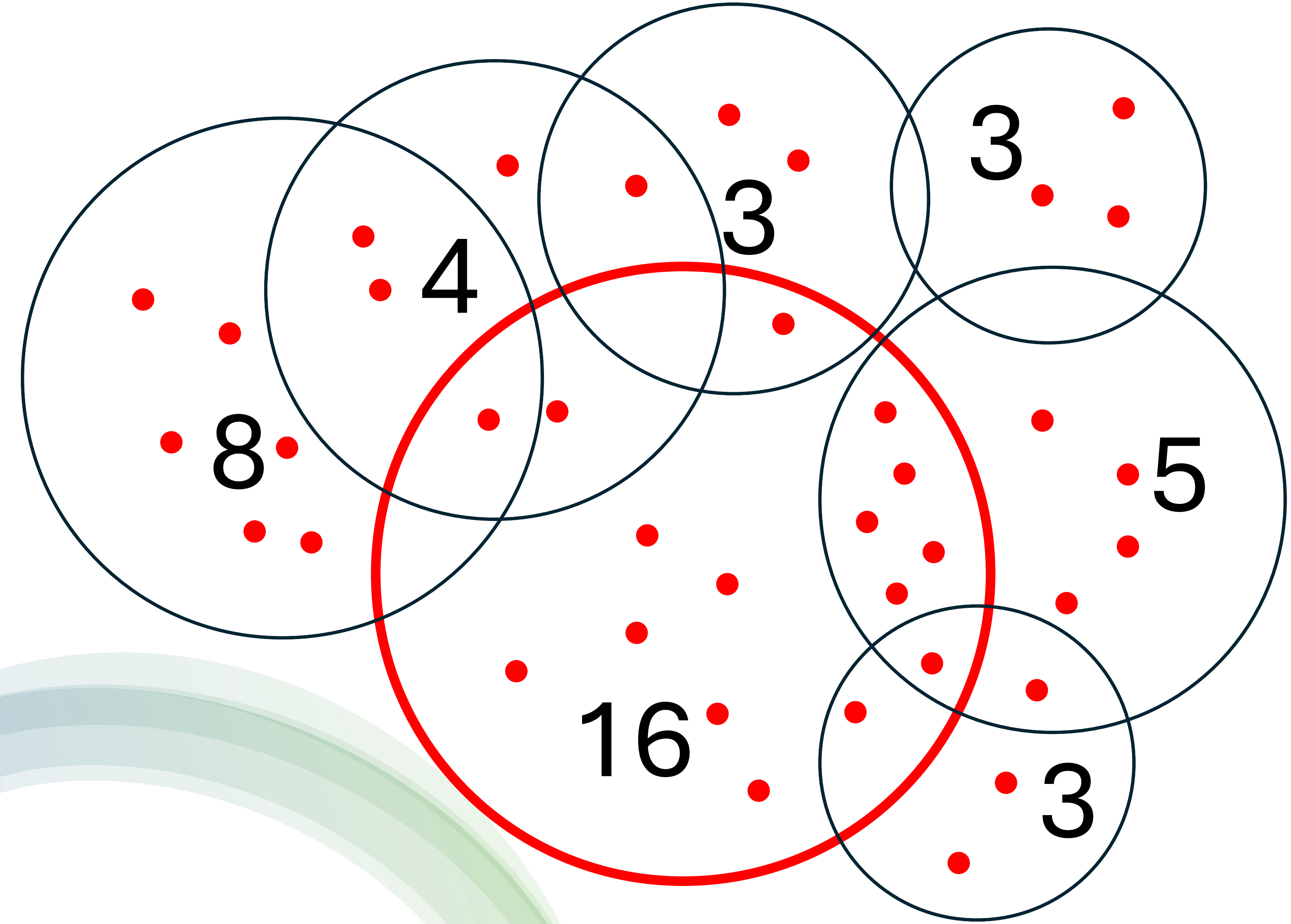

It’s this one!



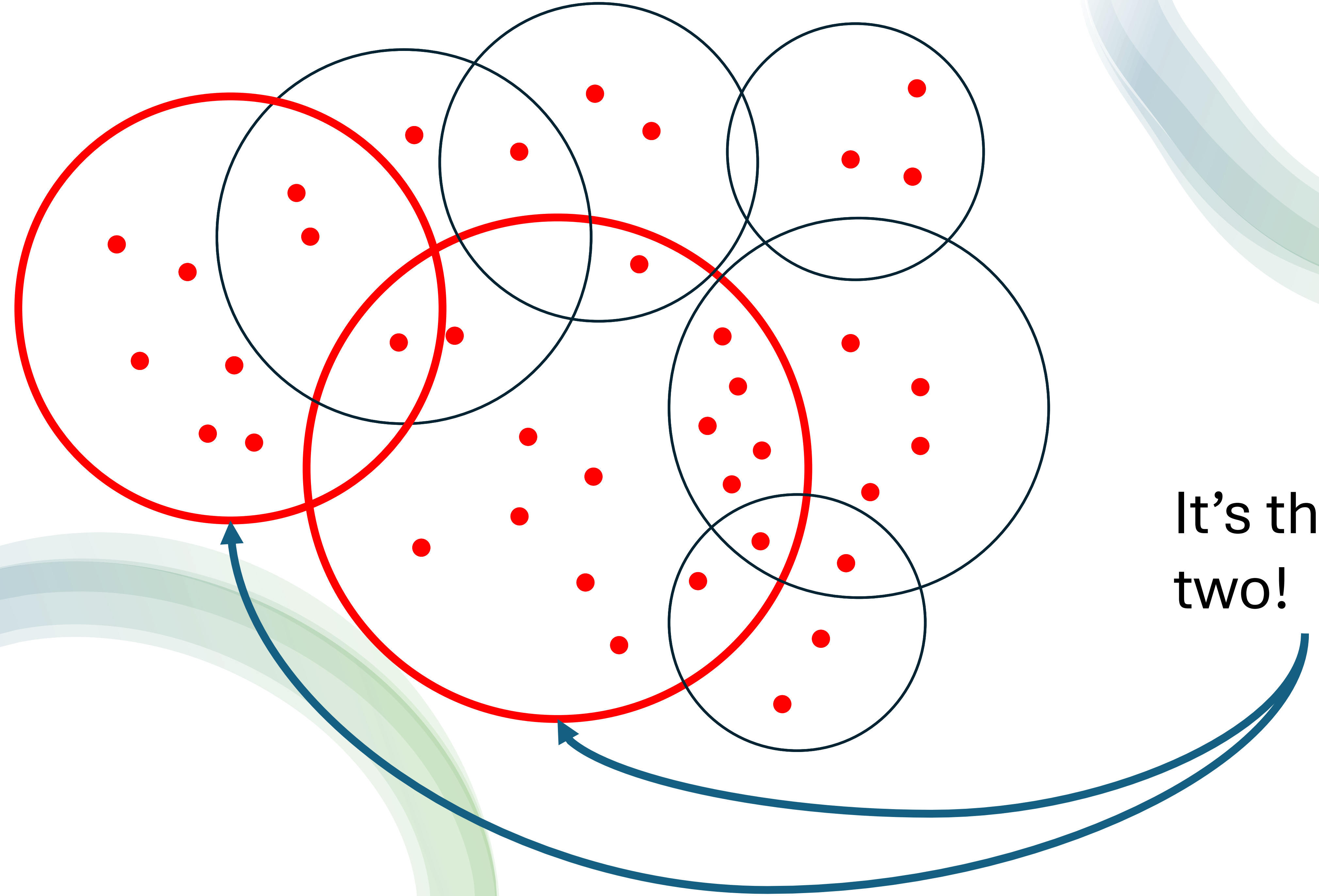
Select the 2 disks that cover the most points.



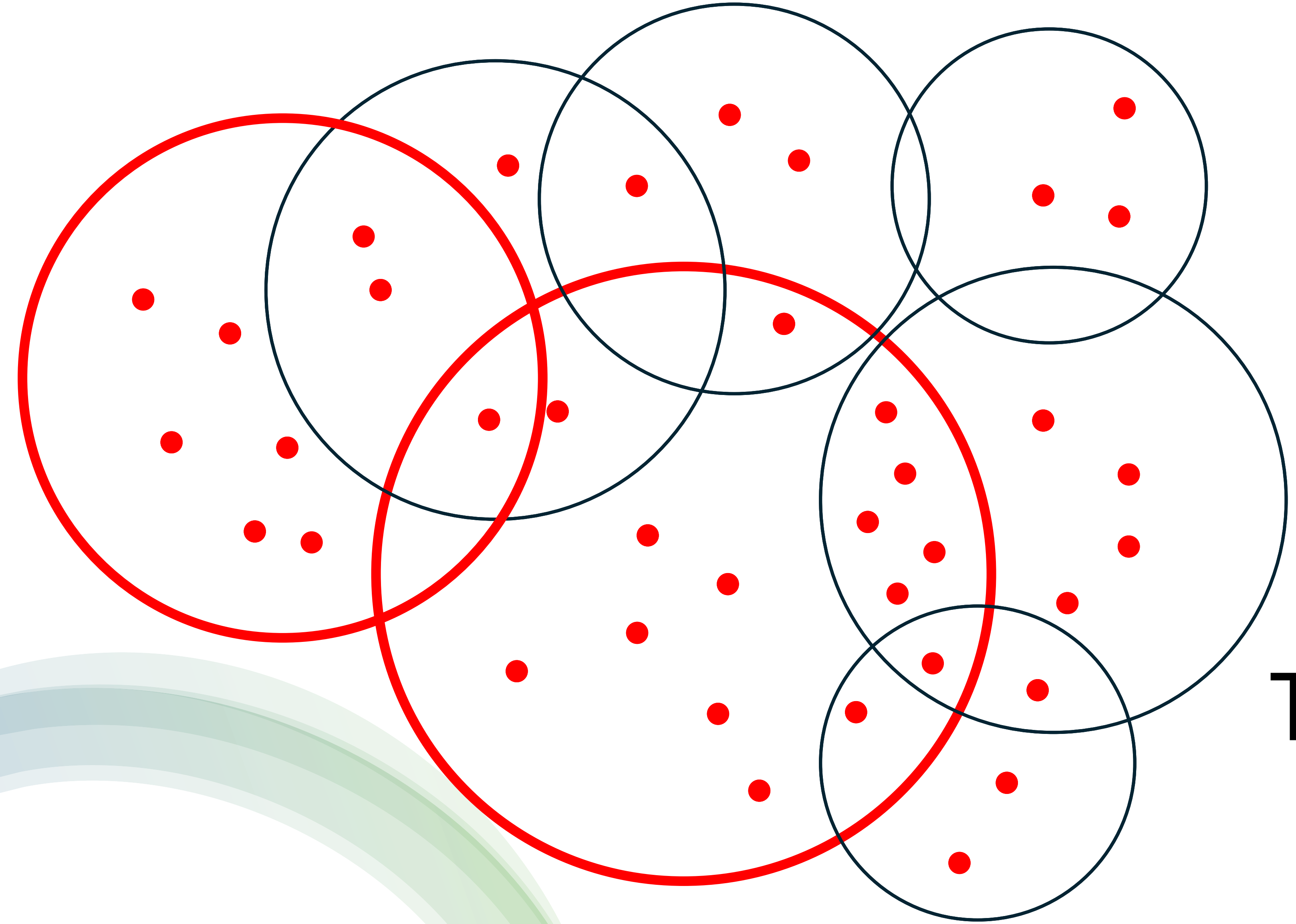
Select the 2 disks that cover the most points.



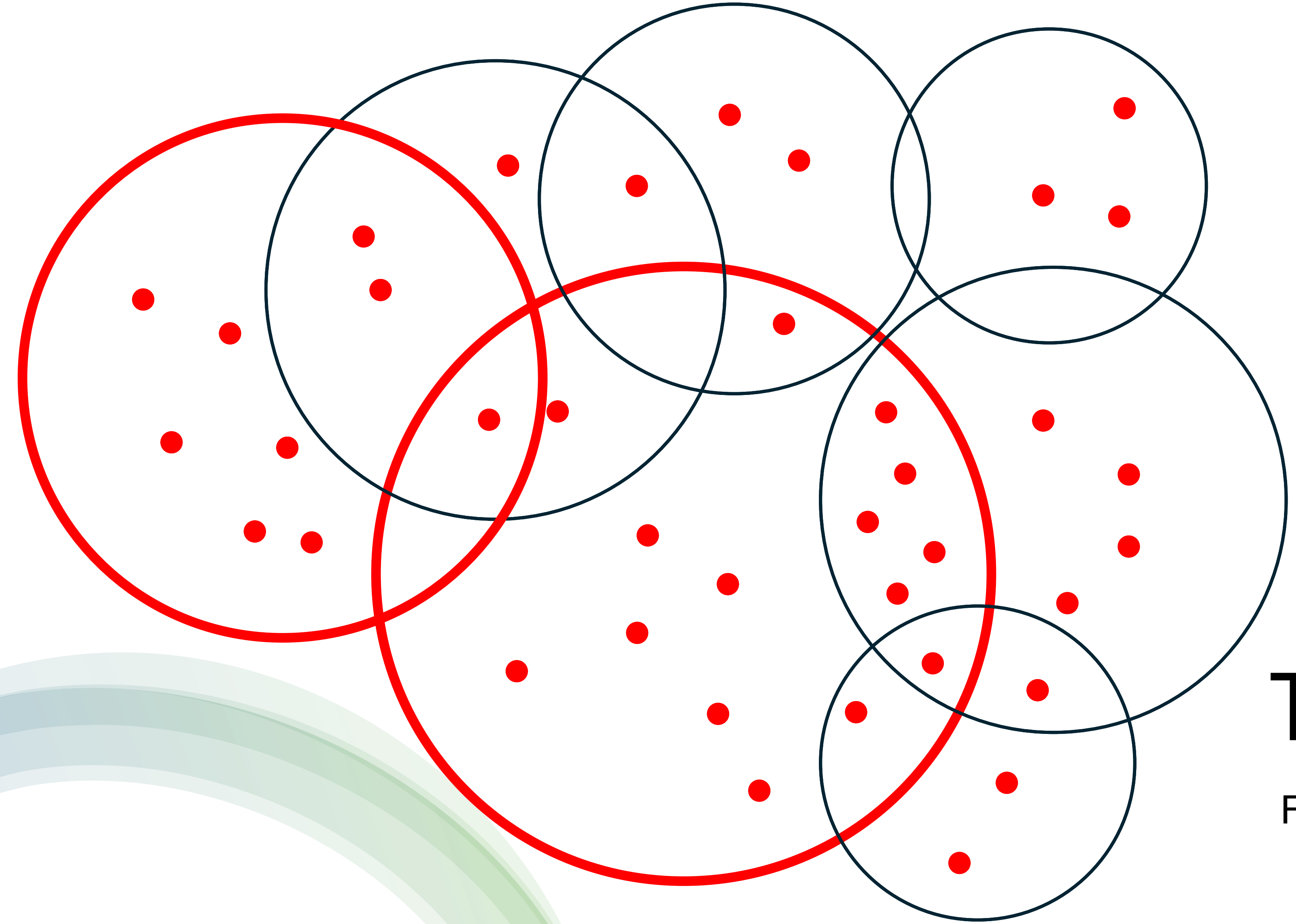
Select the 2 disks that cover the most points.



It’s these two!

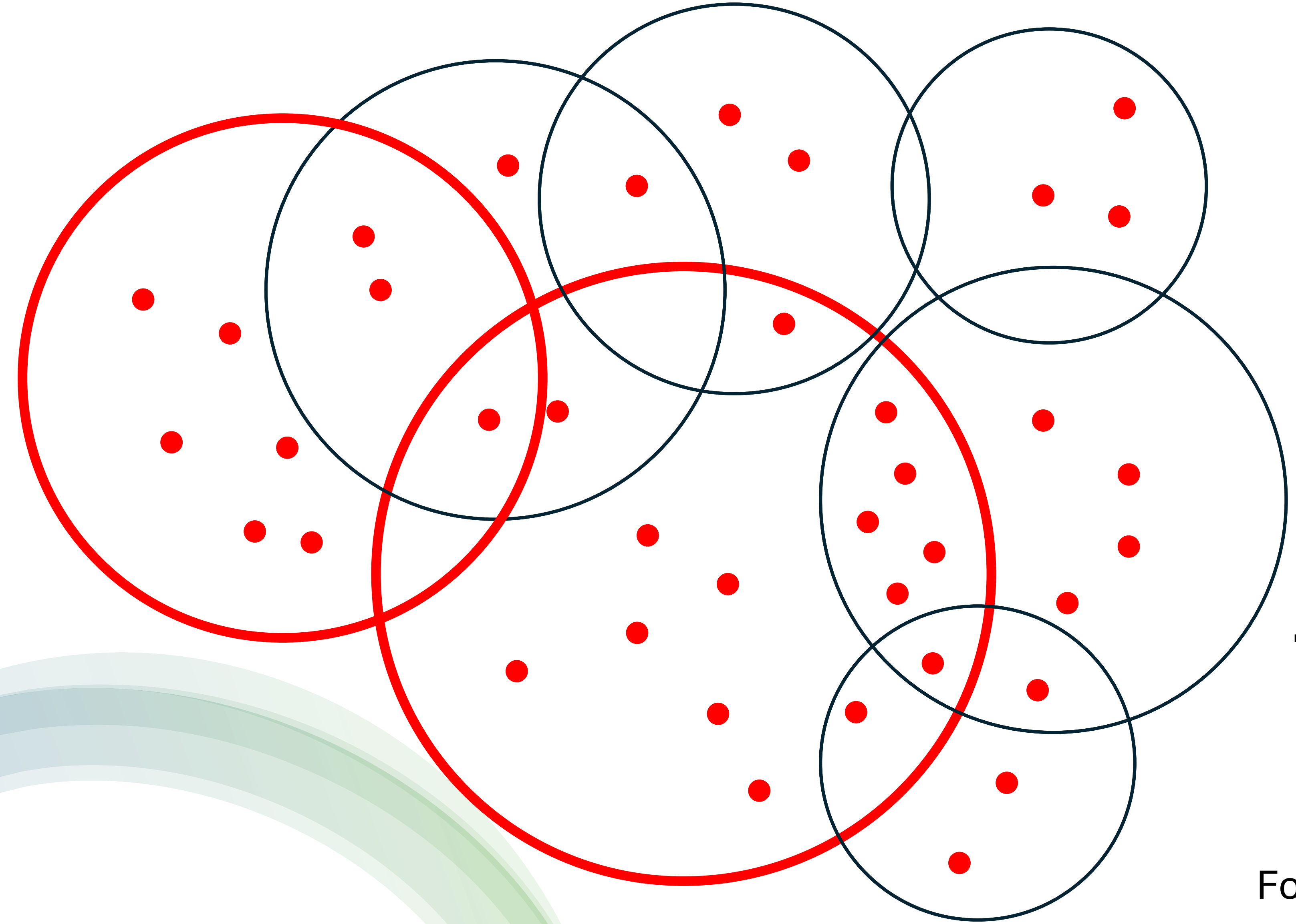


THIS WORKS



THIS WORKS…

For an approximation ratio of $1 - \frac{1}{e}$



There is a PTAS (sometimes)

For any $0 < \varepsilon < 1,\ |A| \geq (1 - \varepsilon)|A*|$

Local-Search

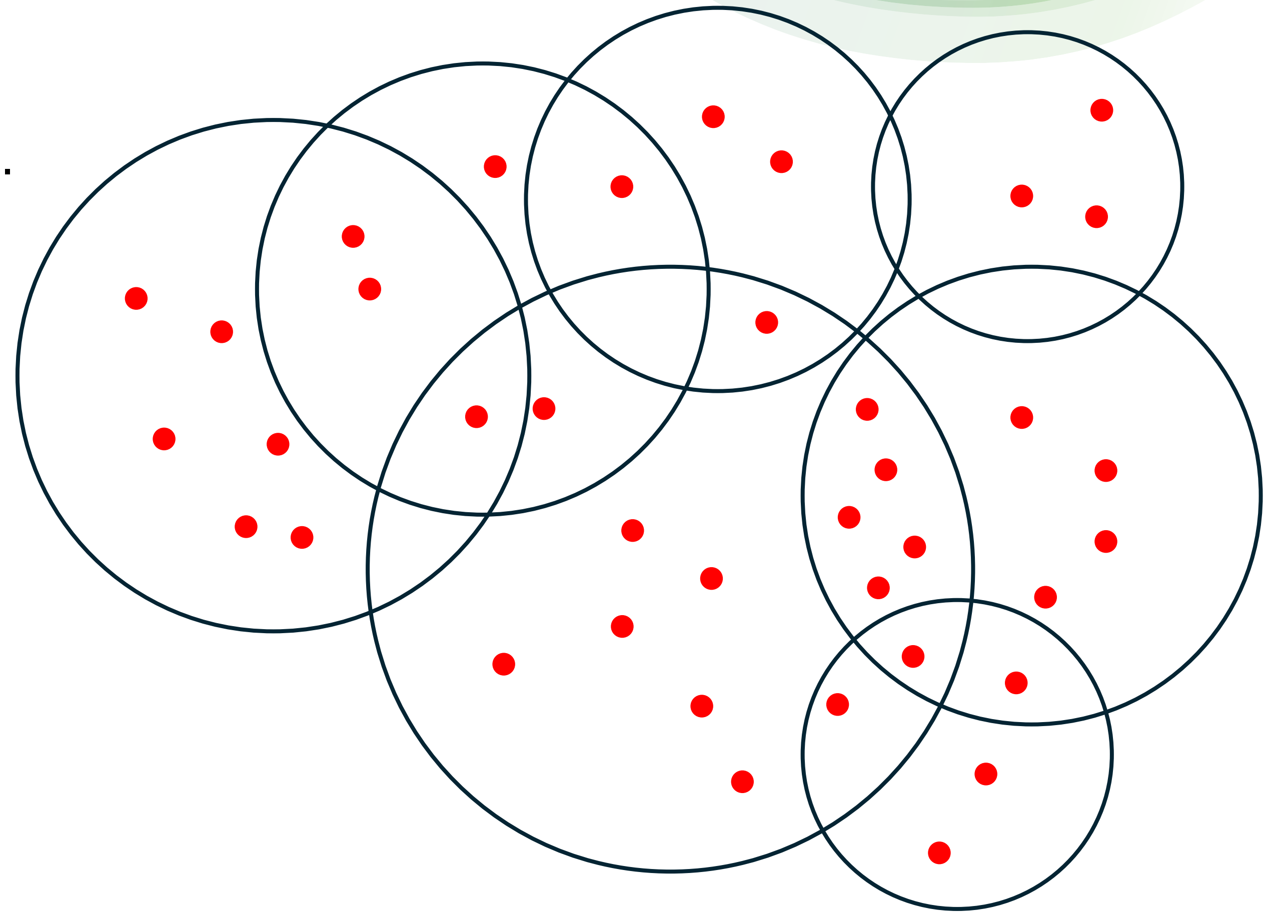
Let ℓ be a constant swap size

Maintain a feasible solution $\mathcal{A} \subseteq \mathcal{F}$ with $|\mathcal{A}| \leq k$.

We repeatedly look for sets $\mathcal{A}' \subseteq \mathcal{A}$ and $\mathcal{B} \subseteq \mathcal{F} \setminus \mathcal{A}$ such that $|\mathcal{A}'| = |\mathcal{B}| \leq \ell$ and $\left|\bigcup \mathcal{A}\right| < \left|\bigcup (\mathcal{A} \setminus \mathcal{A}' \cup \mathcal{B})\right|$.

If such a swap exists, we update $\mathcal{A} \leftarrow (\mathcal{A} \setminus \mathcal{A}') \cup \mathcal{B}$.

Stop when no such ℓ−swap exists.



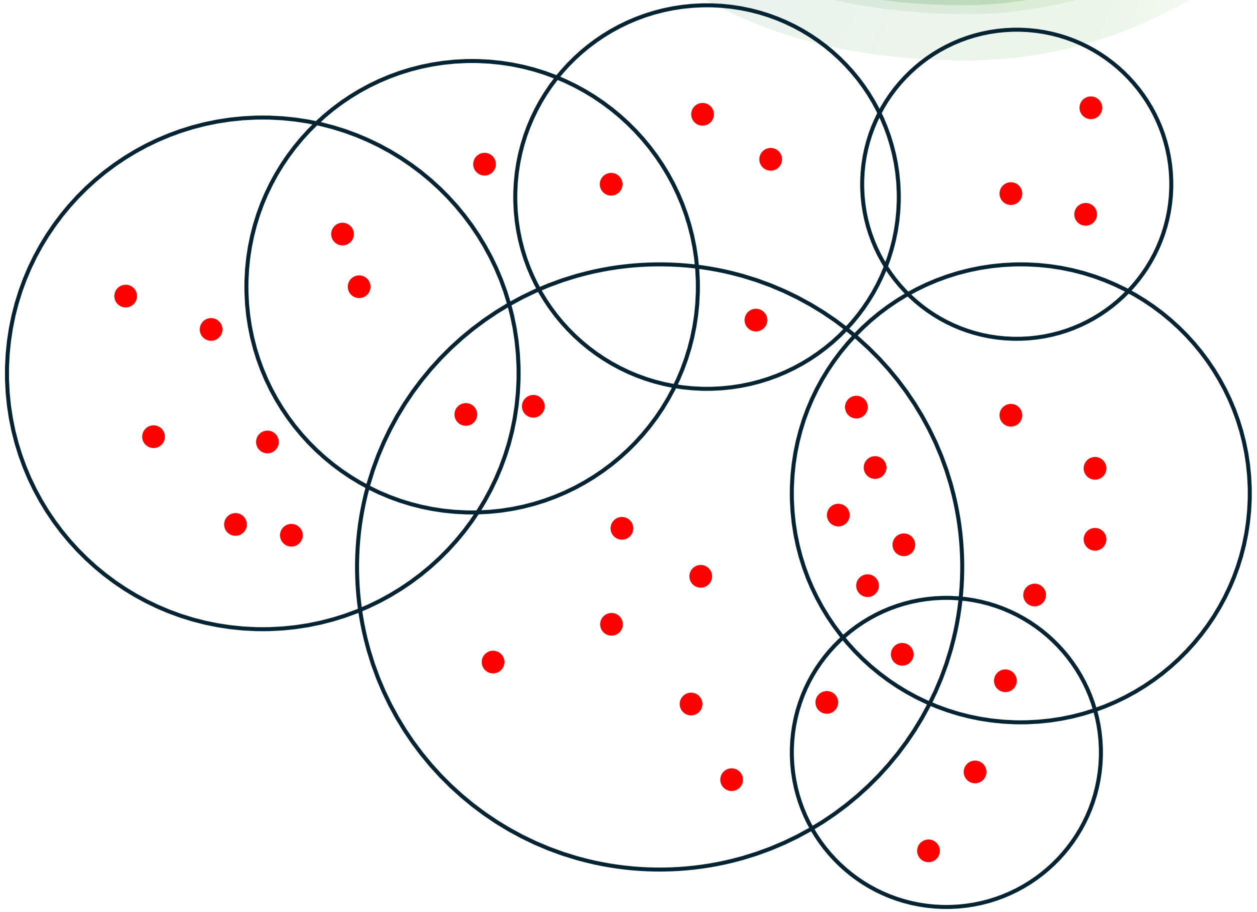
What is ℓ

$$ALG \geq \left(1 - \frac{8c_1 f(b)}{b}\right) OPT \geq (1-\varepsilon) OPT$$

$$\frac{8c_1 f(b)}{b} \leq \varepsilon$$

$$\ell = \Theta(b^2)$$

$$\varepsilon \downarrow \Rightarrow b \uparrow \Rightarrow \ell = \Theta(b^2) \uparrow$$



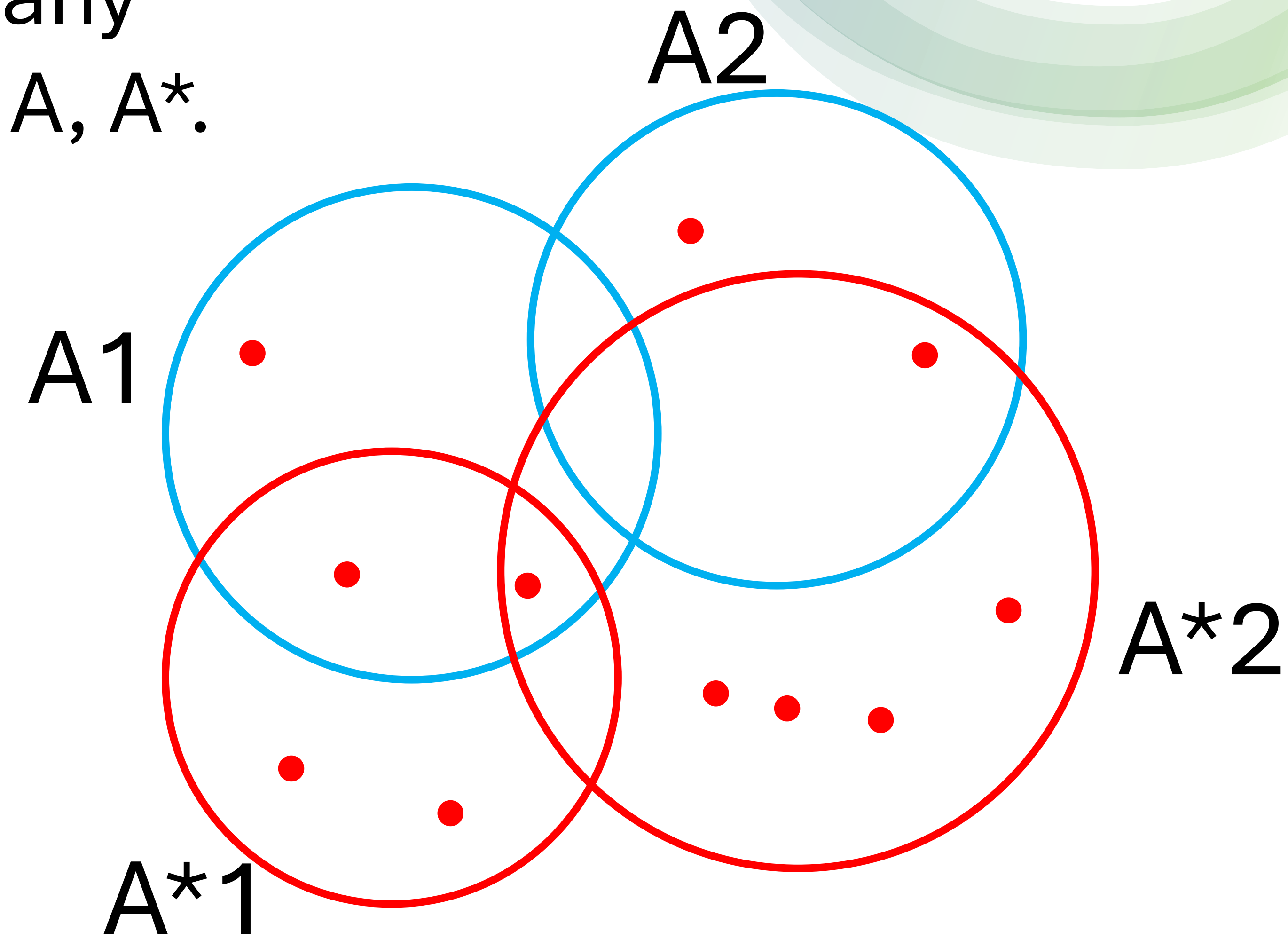
PTAS Proof

For any $0 < \varepsilon < 1,\ |A| \geq (1 - \varepsilon)|A*|$

1. An exchange graph with the exchange property for any two feasible solutions A, A*.

2. We require all such exchange graphs to belong to some f-separable graph class with strictly sublinear f.

An exchange graph with the exchange property for any two feasible solutions A, A*.

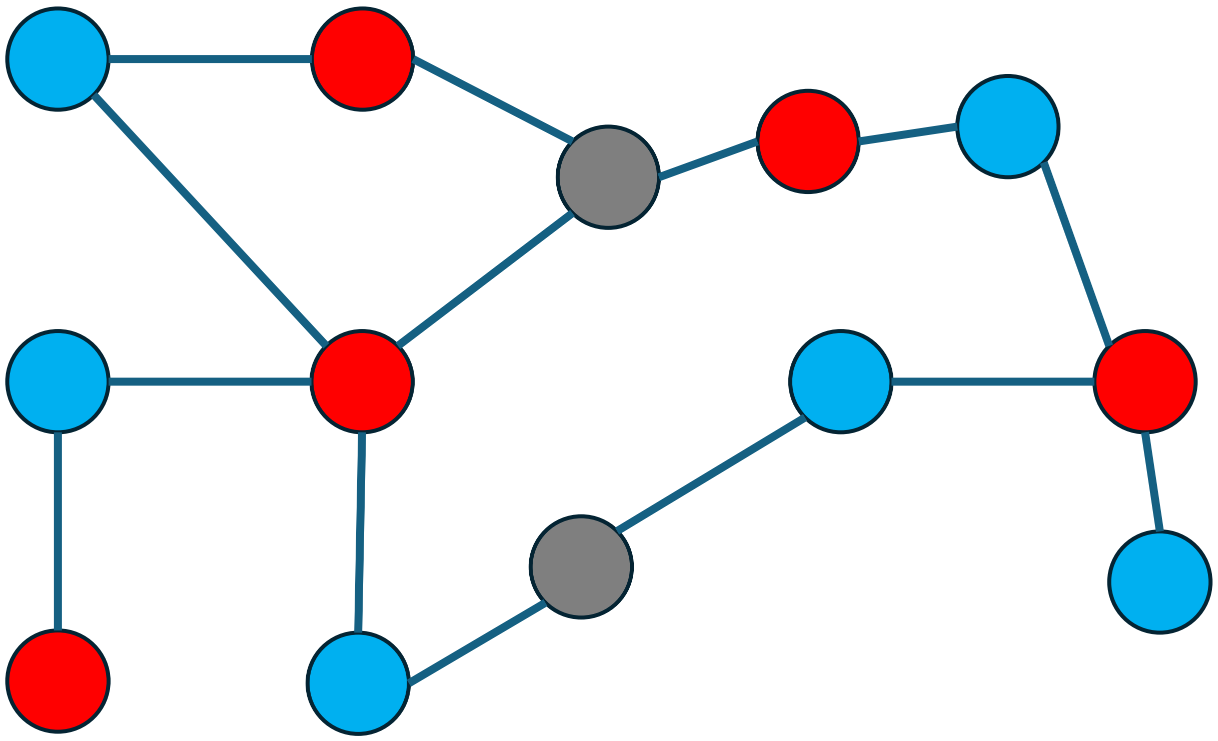


We require all such exchange graphs to belong to some f-separable graph class with strictly sublinear f.

All planar graphs

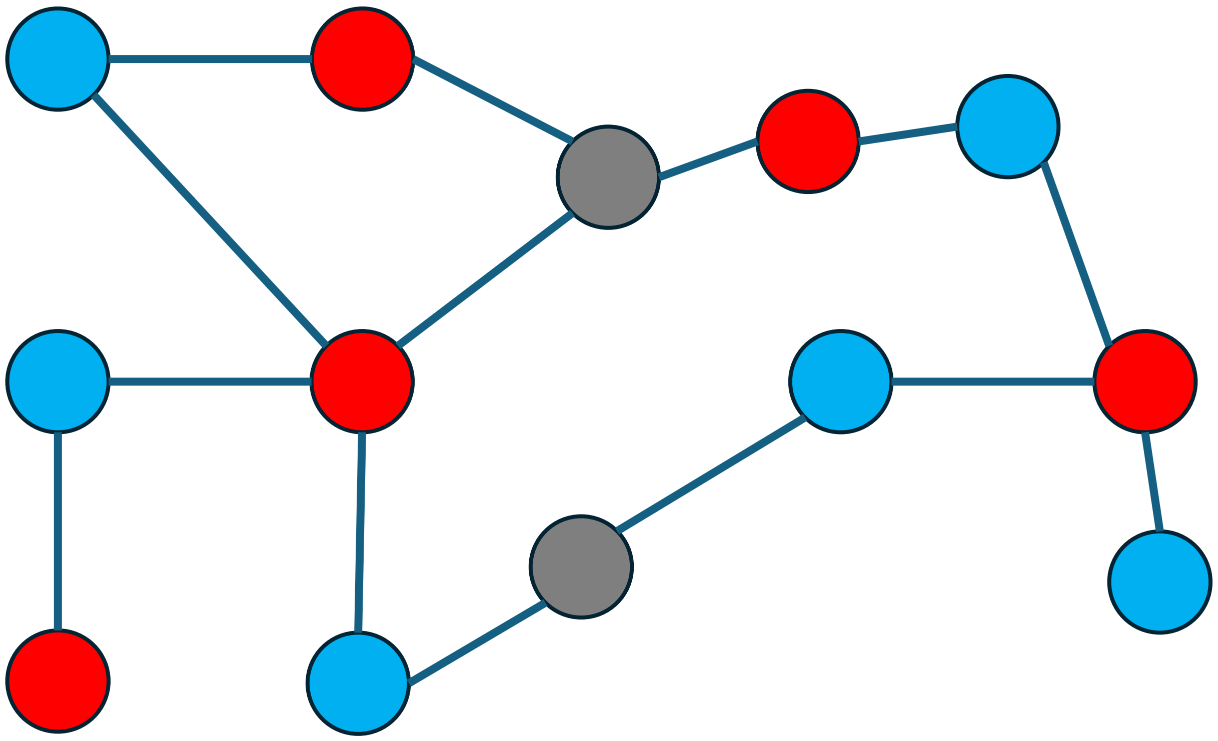
$$f(n) = \Theta(\sqrt{n})$$

And each partition is sized so the local set in each partition is at most ℓ



Colouring

$V_i \;=\; region\ disconnected\ from\ rest\ of\ graph\ by\ X$
$X \;=\; nodes\ chosen\ for\ separation$



$$V(G) = X \,\dot{\cup}\, V_1 \,\dot{\cup}\, \ldots \,\dot{\cup}\, V_t$$

Any edge between V_i and $V_j (i \neq j)$ goes through X.

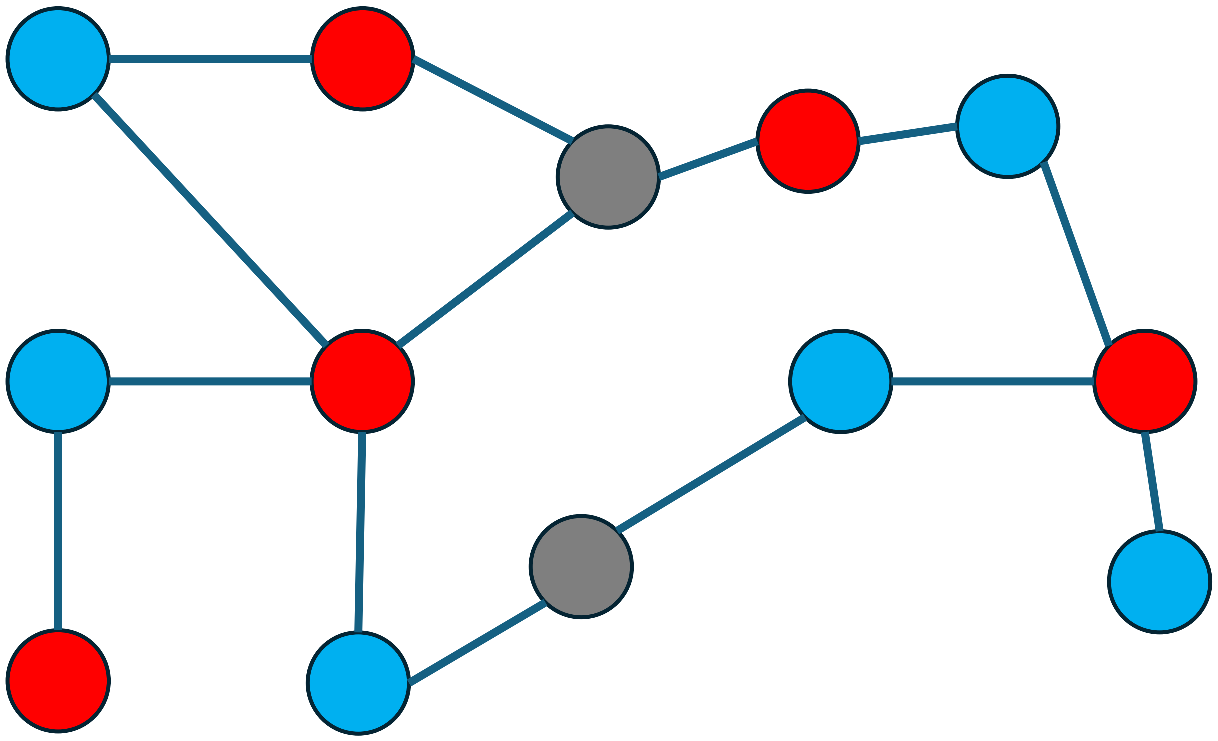
Assumption:

There exists a swap of size $\leq \ell$ that **strictly improves** coverage.

$|L_i|$ = elements lost if we remove A_i

$|W_i|$ = elements gained when we add local optimum

$$\frac{|L_i|}{|W_i|} < 1 - \frac{8c_1 f(b)}{b}$$



Assumption:

There exists a swap of size $\leq \ell$ that **strictly improves** coverage.

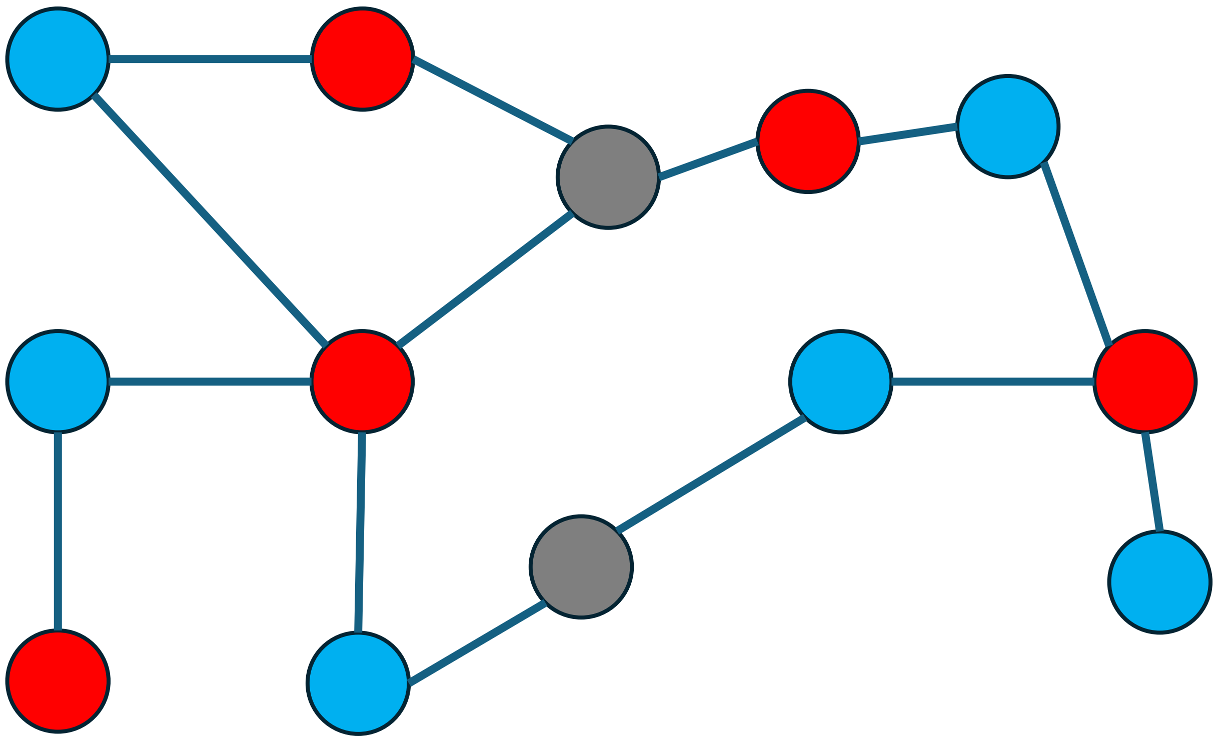
$|L_i|$ = elements lost if we remove A_i

$|W_i|$ = elements gained when we add local optimum

$$\frac{|L_i|}{|W_i|} < 1 - \frac{8c_1 f(b)}{b}$$

We had stated that $ALG \geq \left(1 - \frac{8c_1 f(b)}{b}\right) OPT$

Now we are saying $ALG < \left(1 - \frac{8c_1 f(b)}{b}\right) OPT$



Does that swap exist?

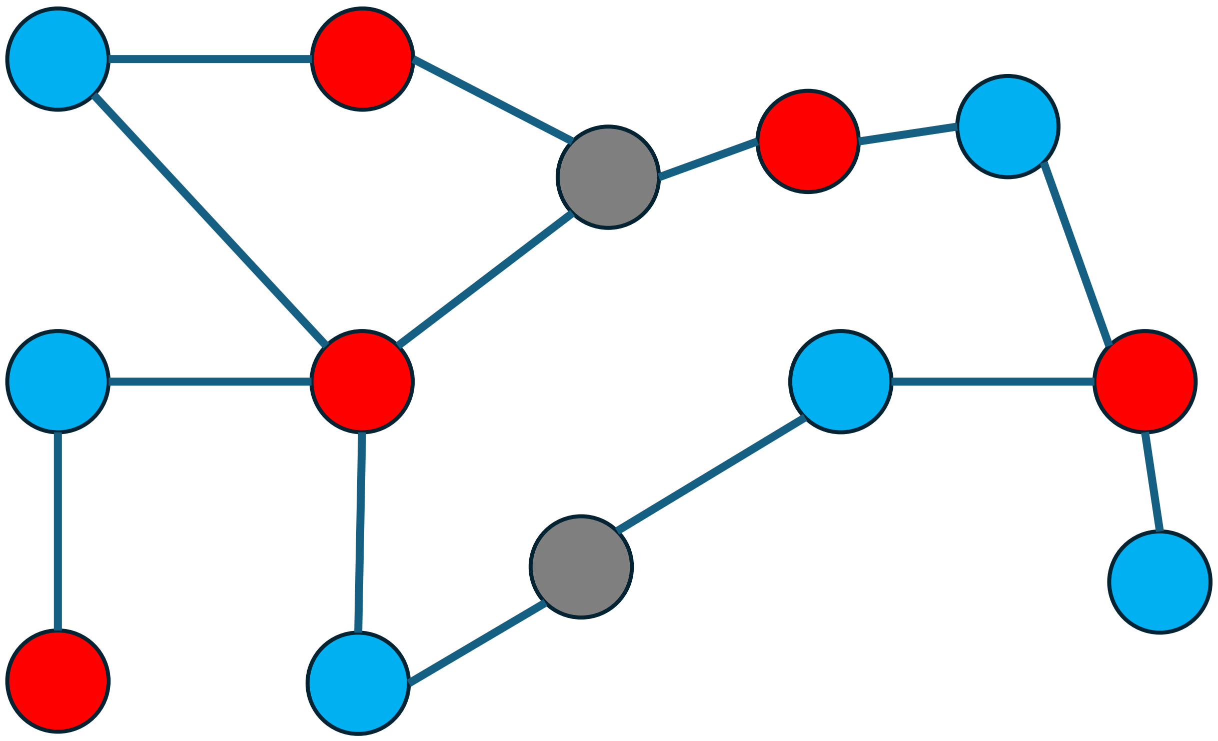
$$\frac{|L_i|}{|W_i|} < 1 - \frac{8c_1 f(b)}{b}$$

If there is a section V_i where there is a locally improving swap

Remove all of A in this section and add |A| from optimal to A

This is a strictly improving ℓ swap since $|A| \leq \ell$

This cannot be proof by contradiction!



Therefore

$$ALG \geq \left(1 - \frac{8c_1 f(b)}{b}\right) OPT$$

Therefore

$$ALG \geq \left(1 - \frac{8c_1 f(b)}{b}\right) OPT$$

$$\mathrm{ALG} \geq (1 - \varepsilon) OPT$$

Therefore

$$ALG \geq \left(1 - \frac{8c_1 f(b)}{b}\right) OPT$$

$$\mathrm{ALG} \geq (1 - \varepsilon) OPT$$

PTAS!

References

Chaplick, S., De, M., Ravsky, A., & Spoerhase, J. (2018). "Approximation schemes for geometric coverage problems." *26th Annual European Symposium on Algorithms (ESA 2018), August 20–22, 2018, Helsinki, Finland*. https://drops.dagstuhl.de/storage/00lipics/lipics-vol112-esa2018/LIPIcs.ESA.2018.17/LIPIcs.ESA.2018.17.pdf

Nabil H. Mustafa, Rajiv Raman, and Saurabh Ray. "Quasi-polynomial time approximation scheme for weighted geometric set cover on pseudodisks and halfspaces." *SIAM J. Comput., https://epubs.siam.org/doi/10.1137/14099317X*

Ashwinkumar Badanidiyuru, Robert Kleinberg, and Hooyeon Lee. "Approximating low-dimensional coverage problems." *In Symp. Computational Geometry (SoCG'12), pages 161–170, 2012. https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.ESA.2018.17*