# [1]Report on
# Sharing the Cost of Multicast Transmission

Deepak Saraswat
Student No: 256741
School of Computer Science
Carleton University
Email: deepak_ saraswat@hotmail.com

**ABSTRACT**
This report discusses cost allocation in the context of multicast transmission. Economic constraints give two distinct mechanisms, *Marginal cost* and *Shapley value*. Marginal cost requires two messages per link of the tree while shapley requires quadratic number of messages in total.

## 1. INTRODUCTION

*Multicast* has long been viewed as an attractive service for the Internet for enabling multiparty applications. In *unicast*, a source has to send a separate copy of the packet to each of its receiver. Which results in a waste of bandwidth as well as increase traffic in the network (which can cause congestion in the network). While in Multicast routing a directed tree is created connecting the source to all of the receiver; when the packet reaches a branch point in the tree, the router duplicates the packet and then sends a copy over each downstream. So, a source can reach to each of its receiver without sending duplicate copies of the packet over any link.

*Pricing multicast* is not very non-intuitive because there is no correlation between the number of participant and the cost for network services. Let us consider a multicast transmission for a set of users $S$ in which each user $i$ has *utility* $u_i$ for receiving the transmission and $C(S)$ is the cost incurred to serve the set of users in S. The function $C$ is known as *Cost sharing function*. Utility $u_i$ of each user $i$ is a private information which is only known to user $i$ .In other words network has to rely on the users to report these values. A user enjoys *welfare* of $u_i - x_i$ (her overall benefit) if she gets the transmission at the price $x_i$.

Therefore, In this type of application we cannot assume that users will report their utility truthfully. They can act selfishly to maximize their own benefits. Thus, one must find a *Cost sharing* mechanism that determines which users receive the multicast transmission and how much they have to pay for that facility. A mechanism is said to be *strategyproof* if the dominant strategy of each user is to reveal the true value of her utility; strategyproofness with few additional constraint described in section 2 leads us to two natural strategy proof mechanism: *Marginal Cost (MC)* and *Shapley Value*[1].

The remainder of this report is organized as follows. Section 2 contains some basic preliminaries. Section 3,4 talks about the Marginal cost mechanism and Shapley value respectively. Section 5 gives the final conclusion.

## 2. MULTICAST TRANSMISSION MODEL

Consider an undirected connected graph $G = (N, L)$, where $N$ is set of the nodes and $L$ is the set of links. P is the set of user and each user $i \in P$ resides at some network location $L$. The cost of the each link $l \in L$ is given by $C(l) > 0$,which is known by the nodes on each end. Multicast flow is emanating from the network node $\alpha_s \in N$. The source node is connected to each user $r \in R$ of the receiver set $R \subseteq P$ through a Multicast tree $T(R) \subseteq L$ rooted at $\alpha_s$. For the paper[2] they have assumed that each user i has a fixed path $T(i)$ from source to it determined by the multicast routing infrastructure. In other words, For the given set of receivers $R$, the multicast tree $T(R)$ is the union of these fixed paths: $T(R) = \cup_{i \in R} T(i)$.

Now there are some economic constraints that have to be satisfied for cost sharing mechanism. These constraints are listed below:

1) **No Positive transfer (NPT):** For each user $i$, $x_i > 0$,users are not paid for receiving the transmission.

[1] This report is based on the paper [2]

2) **Voluntary Participation (VP):** $q_i u_i - x_i > 0$. i.e. users are always free to not receive the transmission, which would result in an individual welfare of Zero.

3) **Consumer sovereignty(CS):**Every user is guaranteed to receive a message if she reports a high enough utility value $u_i$, regardless of the other reported values $u_{-i}$

Instead of the above mentioned constraints, Mechanism must have Submodular, non-negative, non-decreasing Cost function C:

$C(\phi) = 0;\ S \subseteq T \Rightarrow C(S) \leq C(T)$

$C(S \cup T) + C(S \cap T) \leq C(S) + C(T)$ for any $S, T \subseteq P$

There are two other Requirements, Budget balance and Efficiency[1,2], these two are Mutually exclusive, i.e there is no strategyproof cost sharing mechanism which are both budget balanced and efficient.

## 2.1 Computational Model

The primary focus of the paper is communication cost, rather than local computation cost. An instance of the cost sharing problem contains three parameters *(n, p, m)*, where *n* is the number of nodes in the multicast tree, *p* is the number of users (population size) and m is the total size of the input: $\{C(l)\}_{l \in L} \cup \{u_i\}_{i \in P}$

Some of the major issues for communication – complexity includes that the total messages on links should be $O(n)$ and Maximal number of messages on link should be $O(1)$. The paper [2] ignores the issues of Local computation complexity and maximal message size.

# 3. MARGINAL COST MECHANISM:

**Theorem 1:** The marginal cost sharing algorithm require exactly two messages per link.

**Proof:** Marginal cost algorithm computes the cost shares by performing one bottom –up traversal on tree, followed by one top down traversal. In order to describe the algorithm we need the following notation. Let $u^{\alpha}$ is the sum of the utilities of the users located at node $\alpha$, *c* is the cost of the link between $\alpha$ and its parent, $Ch(\alpha)$ is all the child of $\alpha$ in the tree, $V(P)$ is all the nodes in the tree $T(P)$ and $W^{\alpha}$ is the welfare from the subtree rooted at $\alpha$, and its defined as follows:

$$W^{\alpha}(u) = u^{\alpha} + \left[\sum_{\beta \in Ch(\alpha) | W^{\beta}(u) \geq 0} W^{\beta}(u)\right] - c^{\alpha} \quad (1)$$

At Node $\alpha \in V(P)$

After receiving a messages $A^{\beta}$ from each child $\beta \in Ch(\alpha)$

$W^{\alpha} \leftarrow u^{\alpha} + \left[\sum_{\beta \in Ch(\alpha) | W^{\beta}(u) \geq 0} W^{\beta}(u)\right] - c^{\alpha}$

If $W^{\alpha} \geq 0$ then

{

$q_i \leftarrow 1$ for all $i \in res(\alpha)$

}

Else {

$q_i \leftarrow 0$ for all $i \in res(\alpha)$

Send $0$ to parent $P(\alpha)$

}

**Figure A:** Bottom up traversal: Computing Welfare values

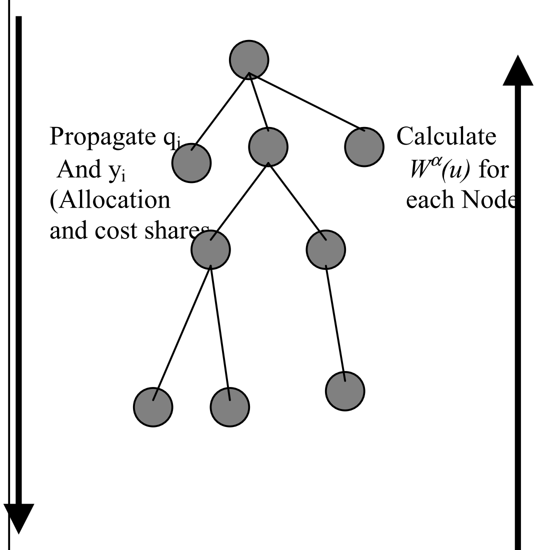


Figure b: Total of exactly 2 messages per link.

According to VCG[1] formula the cost shares is given by:

$$x_i(u) = u_i q_i - [W(N,u) - W(N-i,u)]$$

And

$$W(N, u) = \max_{T \subseteq N}[u_T - C(R(T))]$$

Now the problem is how to compute $W(N-i, u)$. Let $y_i(u)$ be the smallest $W^{\beta}(u)$ in the path from $\alpha$ to the root. Then we have to take care of the following cases:

**Case 1**: If $u_i \le y_i(u)$, then the receiver set remain the same when dropping user *i*. Thus, $W(N, u)-W(N-i, u)=u_i$. So the user i pay $x_i(u)=u_i-[W(N,u)-W(N-i,u)]=0$

**Case 2**:If $u_i>y_i(u)$, Then dropping user i result in the elimination of the subtree with the total welfare $y_i$ (u). So the user i must pay $x_i(u)=u_i-[W(N, u)-W(N-i, u)] = u_i-y_i(u)$.

**Figure b** shows how the algorithm requires only two messages per link.

-The $W^{\alpha}(u)$ is computed by bottom up traversal, while the allocation and cost shares is computed by the same top down traversal.

Initialize: Root $\alpha_s$, sends $W^{\alpha}_s$ to each of its children
For each $\alpha \in V(P)-\{\alpha_s\}$
After receiving a message A from parent p ($\alpha$)

//case 1: $T^{\alpha}(P) \cap T(R^*(u))=\phi$
//Set qi properly and propagate non-
//membership downward
If $q_i = 0$,for all $i \in res(\alpha)$,or $A<0$ then
{
$x_i \leftarrow 0$ and $q_i \leftarrow 0$ for all $i \in res(\alpha)$
send $-1$ to $\beta$ for all $\beta \in Ch(\alpha)$
}

//Case 2: $T^{\alpha}(P) \cap T(R^*(u)) \neq \phi$
//Compute cost shares and propagate
//minimum welfare value downward
Else
{
$A \leftarrow \min(A, W^{\alpha})$
For each $i \in res(\alpha)$
If $u_i \le A$, then $x_i \leftarrow 0$, else $x_i \leftarrow u_i-A$
For each $\beta \in Ch(\alpha)$
Send $A$ to $\beta$
}

**Figure c**: Top-Down traversal: Computing Membership Bits and cost shares

# 4. SHAPLEY VALUE MECHANISM

The Shapley cost sharing mechanism Distributes the cost of each edge equally among all of the users located downstream of the edge. The simplest case of the *SH* cost share is the one in which all $u_i$ are sufficiently large to guarantee that all of P receives the transmission. Cost share for Shapley value is computed as follows:

**Step 1:**Do a bottom up traversal of the tree that determines, the number of users ($p_\alpha$) in the subtree rooted at each node $\alpha$.

**Step 2**:Do a Top down traversal, which the root initiates by sending the number *md=0* to all its children. After receiving message md, node $\alpha$ computes $md' \equiv (c(l)/p) + md$, where $l$ is the network link between $\alpha$ and its parent, assigns the cost share md′ to each of its resident users, and sends md′ to each child.

So each user ends up paying a fraction of the cost of each link in its path from the source and the fraction is determined by the numbers of users sharing the link.

## Theorem:

Shapley's cost sharing requires, in the worst case, $\Omega(n \cdot p)$ message exchanges ($\Omega(n^2)$ when $p=O(n)$ )

## Proof:

Consider a multicast tree with nodes $\alpha_o$ through $\alpha_n$. Node $\alpha_o$ acts as a source and the cost of the $[\alpha_i, \alpha_{i+i}]$ is $c_{i+1}$ which is only known to node $\alpha_i$. Each of node $i=1\ldots n-1$ has an agent with utility $u_i$ while there are $n$ agent at $\alpha_n$, with utilities $u_{nj}$. Suppose that utilities $u_i$ are high enough and we have to check for users at $\alpha_n$ and exclude one by one at each iteration of the brute force algorithm. When the $u_{nj}$ are in the decreasing order, this is a tantamount to checking the inequalities

$$u_{nj} < \sum c_i \,/\, (n+j-i) \qquad (2)$$

As we are only interested in the message across link $[\alpha_{n-1}, \alpha_n]$, we can regard the network as two nodes $\alpha_n$ and $\alpha$ (formed by the merging of rest of nodes). The $u_{nj}$ is local to $\alpha_n$ while $c_i$ are known to the node $\alpha$. The claim of the theorem follows from the following Lemma of distributed computation.

**Lemma 1.** Any linear distributed algorithm by two nodes for checking the inequalities $Ax+By > b$, where x is an n-vector known to node *I*, *y* is an *n*-vector known to *II*, *A*, *B* are $n \times n$ nonsingular Matrices known to both, and *b* is an *n*-vector known to both, requires n message exchanges.

If we applied the above lemma in the inequality (2).with *x* standing for the $u_{nj}$'s and *y* for the $c_i$'s,

$A$ = $-I$ and $B$ the non singular matrix, we find that at least $n$ messages must be exchanged across link $[\alpha_{n-1}, \alpha_n]$ and $n-1$ messages across link $[\alpha_{n-2}, \alpha_{n-1}]$, in general $n-j$ messages are needed across link $[\alpha_{n-j-1}, \alpha_{n-j}]$. It shows that $\Omega(n^2)$ messages are needed in total.

## 5. Conclusion

*The* MC mechanism is implementable with an algorithm that only requires a single message sent in each direction on each link in the *tree T (P).* While *SH* mechanism requires a linear number of messages on a linear number of links. Which is roughly the same amount of communication used by the centralized approach of sending all the $u_i$ and $c_j$ values to a designated node, computing the resulting cost shares at that designated node, and then sending the $x_i$ and $q_i$ values back to each node. This centralized approach can be applied to all polynomial-time cost- sharing mechanisms; therefore, the Shapley value has no benefit for being distributed. So the two mechanisms are at the opposite ends of the feasibility spectrum.

**Bibliography**


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