

KRUSKAL MST($G=(V,E); w:E \rightarrow \mathbb{R}^+$)

Let $V := \{x_1, x_2, \dots, x_n\}; E = \{e_1, e_2, \dots, e_m\}$

1. SORT E w.r.t weight.
WLOG, Let $w(e_1) \leq w(e_2) \leq w(e_3) \leq \dots \leq w(e_m)$

2. For $i := 1$ to n do $\left[\begin{array}{l} V_i := \{x_i\}; \text{Label}[x_i] := i; \\ \text{size}[V_i] := 1; T_i := \emptyset; \end{array} \right.$

3. For $k := 1$ to m do [edges in sorted order]

Let $e_k = (p_k, q_k);$



$i := \text{Label}[p_k]; j := \text{Label}[q_k];$

If $i \neq j$ then {Edge connects two forests}

If $\text{size}[V_i] \geq \text{size}[V_j]$ then

$\left[\begin{array}{l} \text{For each vertex } z \in V_j: \boxed{\{V_i := V_i \cup V_j\}}; \\ \text{Label}[z] := \text{Label}[p_k]; \\ \text{size}[V_i] = \text{size}[V_i] + \text{size}[V_j]; \\ T_i := T_i \cup T_j \cup \{e_k = (p_k, q_k)\} \\ T_j := \emptyset; \\ V_j := \emptyset; \end{array} \right.$

else $\left[\text{repeat with roles of } i \text{ \& } j \text{ reversed.} \right.$

4. Return (T)

PRIM MST ($G=(V,E), w:E \rightarrow \mathbb{R}^+$)

1. $r :=$ Arbitrary vertex of V ;

$A := \{r\}$; $T := \emptyset$;

2. INITIALIZE $\forall y \in V \setminus \{r\}$ do $\left[\begin{array}{l} \text{minwt}(y) := \infty; \\ \text{closest}(y) := \text{NIL}; \end{array} \right.$

For each edge $(r,y) \in E$ do

$\left[\begin{array}{l} \text{minwt}(y) := w(r,y); \\ \text{closest}(y) := r; \end{array} \right.$

3. $Q := V \setminus \{r\}$; $k := 1$;
(Q is a min-heap w.r.t. minwt)

4. While $k \neq n$ do

$\left[\begin{array}{l} v := \text{Extract-MIN}[Q]; \quad u := \text{closest}(v); \\ A := A \cup \{v\}; \quad Q := Q \setminus \{v\}; \\ T := T \cup \{(u,v)\}; \quad k := k+1; \\ \text{For each edge } (v,y) \in E \text{ do} \\ \quad \left[\begin{array}{l} \text{if } y \in Q \text{ and } w(v,y) < \text{minwt}(y) \text{ then} \\ \quad \left[\begin{array}{l} \text{minwt}(y) := w(v,y); \\ \text{closest}(y) := v; \end{array} \right. \end{array} \right. \end{array} \right.$

5. Return (T)