Problem 1. Formulate a decision problem corresponding to the following optimization problems:

1. (Clique) In an undirected graph $G = (V, E)$, find the largest size clique. A set of vertices $K \subseteq V$ is said to form a clique, if for every pair of vertices $u, v \in K$, $uv \in E$.

2. (Independent Set) In an undirected graph $G = (V, E)$, find the largest size independent set. A set of vertices $I \subseteq V$ is said to be independent, if for every pair of vertices $u, v \in I$, $uv \not\in E$.

3. (Vertex Cover) In an undirected graph $G = (V, E)$, find the smallest size vertex cover. A set of vertices $C \subseteq V$ is said to form a cover, if for every edge $e = (u, v) \in E$, $u \in C$ or $v \in C$.

Problem 2. For the above three decision problems, state an equivalent formulation in terms of the language of the decision problem.

Problem 3. Let $k < n$ be a positive integer. Let us define the language $k$–COMP = \{ $G = (V, E)$, $G$ is a simple undirected graph on $n$ vertices containing at most $k$ components \}. Is $k$–COMP $\in$ P? Is $k$–COMP $\in$ NP?

Problem 4. Are the decision version of the problems stated in Problem 1 in NP?

Problem 5. Let $n = |V|$. Show that the decision problems stated in Problem 1 are equivalent with respect to the polynomial time reducibility. I.e., show that

1. Clique($G, k$) $\leq_P$ Independent-set($\bar{G}, k$)

2. Independent-set($\bar{G}, k$) $\leq_P$ Vertex-Cover($\bar{G}, n - k$)

3. Vertex-Cover($\bar{G}, n - k$) $\leq_P$ Clique($G, k$)

Problem 6. Show that each of the decision problems stated in Problem 1 are NP-complete.

Problem 7. Let $S$ be a set of $n$ positive numbers and let $t$ be an integer value. Define the language Subset-Sum($S, t$) as follows:

Subset-Sum($S, t$) = \{ All sets $S'$ of $n$-numbers s.t. $\exists S' \subseteq S$, where $\sum_{x \in S'} x = t$ \}.

Show that Subset-Sum($S, t$) is NP-complete.
References

