Carleton University

Name: A.M.	Problem Set: 3
Course: COMP 3804	Week: 1-3

Problems

Consult [1, 2, 3].

Problem 1 (Exercise 2.32 of [2]). Let S be a set of n points on the real line. How fast can you find a pair of points that have the smallest distance? What if the points are in 2-dimensional real plane.

Problem 2 (Section 4.1 of [1]). Let A be an array of size n, where each A[i] is an integer (positive or negative) and $1 \le i \le n$. For $1 \le i \le j \le n$, define $\Delta(i, j) = \sum_{k=i}^{j} A[k]$. Find a maximum subarray of A, i.e., find a pair of indices α and β , where $1 \le \alpha \le \beta \le n$, such that $\Delta(\alpha, \beta) \ge \Delta(i, j)$, for all possible choices of $1 \le i \le j \le n$. Devise first a naive algorithm running in $O(n^3)$ time by considering all possible choices of i and j. Devise an $O(n \log n)$ time divide-and-conquer algorithm. Can you come up with an algorithm running in O(n) time? See Exercise 4.1-5 of [1].

Problem 3. Let A be an array consisting of n distinct integers. Show that we can compute the maximum and the minimum element of A by performing at most 3n/2 comparisons.

Problem 4. Let A be an array consisting of n distinct integers. Show that we can compute the maximum and the second maximum element of A by performing at most $n + \log n$ comparisons.

Problem 5. (Problem 9.1-2 of [1]) Let A be an array consisting of n distinct integers. Show that we need to perform $\lceil \frac{3n}{2} \rceil - 2$ comparisons in the worst case to find simultaneously the maximum and the minimum element of A.

Problem 6. Let A be an array consisting of n distinct real numbers. Assume that you can find the median of A in O(n) time. How can use the median finding algorithm to find the k-th largest element for any given value of k, where $1 \le k \le n$.

Problem 7. In the deterministic median finding algorithm, we partitioned the input elements to form groups of size 5. We showed that the running time of the algorithm with this partitioning is O(n). What will be the running time if we form groups of size:

(a) 7

(b) 3

Problem 8. Let x be a number. Let $n = 2^k$ for some positive integer k. Present an algorithm running in $O(\log n)$ time to compute: $z = x^n \mod 10$.

References

- T. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to Algorithms*. 3rd. ed., MIT Press, 2009.
- [2] S. DasGupta, C. Papadimitriou, V. Vazirani. Introduction to Algorithms. McGraw Hill.
- [3] A. Maheshwari. *Notes on Algorithm Design*, Chapter 1, https://people.scs.carleton.ca/~maheshwa/Notes/DAA/notes.pdf