School of Computer Science

Carleton University

Name: **A.M.** Problem Set: **6**
Course: **COMP 3804**

Problems

Consult [1, 2, 3]. These exercises are based on dynamic programming. Recall the three steps:

1. Optimal Substructure - Optimal solution contains optimal solutions for smaller sub-problems.
2. Setting up recurrence relation
3. Evaluate the recurrence bottom up - solve smallest sub-problems first, and then increasing larger sizes.

Problem 1. *(Input consists of $x_1, \ldots, x_n$ and subproblem is $x_1, \ldots, x_i$.) Find a longest path in a DAG.*

Problem 2. *(Input consists of $x_1, \ldots, x_n$ and subproblem is $x_1, \ldots, x_i$.) Given a sequence of distinct integers $x_1, \ldots, x_n$, find a longest increasing subsequence.*

Problem 3. *(Input consists of $x_1, \ldots, x_n$ and subproblem is $x_1, \ldots, x_i$.) A bag can hold at most W kilos, where W is a positive integer. You are given an unlimited supply of n-different items. Each item i has an integer weight w_i kilos and its value is v_i. We need to fill the bag with the items to maximize the sum total of the values of the items in the bag but at the same time not to exceed the weight capacity of the bag. Design a DP-based algorithm whose running time is $O(Wn)$.*

Problem 4. *(Input consists of $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$ and the subproblem is $x_1, \ldots, x_i$ and $y_1, \ldots, y_j$.) Given sequences $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_m)$ find the longest common subsequence.*

Problem 5. *(Input consists of $x_1, \ldots, x_n$ and the subproblem is $x_i, \ldots, x_j$) Let P be a set of n points in convex position in the plane. Find a triangulation of P so that the total perimeter of all the triangles in the triangulation is minimized.*

Problem 6. *(Input consists of a rooted tree and the sub-problem is a subtree) Let $G = (V, E)$ be a simple undirected graph. A subset of vertices $I \subseteq V$ is said to be independent if there is no edge in the graph between a pair of vertices in I. Find the largest independent set of a tree.*

References

[1] T. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to Algorithms.* 3rd. ed., MIT Press, 2009.

[2] S. DasGupta, C. Papadimitriou, V. Vazirani. *Introduction to Algorithms.* McGraw Hill.

[3] A. Maheshwari. *Notes on Algorithm Design*, Chapter 1, `https://people.scs.carleton.ca/~maheshwa/Notes/DAA/notes.pdf`