School of Computer Science

Carleton University

Name: Problem Set: **1**

Course: **COMP 3804**   Week: **1**

Problems

Consult references [1, 2].

Problem 1. *Define O, Ω, Θ in the context of this course.*

Problem 2. *Applying the definitions, show the following:*

1. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

2. $1 + 2 + 3 + \cdots + n \in O(n^2)$.

3. $1 + 2 + 3 + \cdots + n \in \Omega(n^2)$.

4. $1 + 2 + 3 + \cdots + n \in \Theta(n^2)$.

Problem 3. *Show that $\sum_{k=1}^{n} k^2 \in \Theta(n^3)$*

Problem 4. *Show that $\frac{1}{3}n^2 - 3n \in O(n^2)$*

Problem 5. *Let $p(n) = \sum_{i=0}^{d} a_i n^i$ be a polynomial of degree d and assume that $a_d > 0$. Show that $p(n) \in O(n^k)$, where $k \geq d$ is a constant. What are c and n_0 if we use the following definition of O-notation: $O(g(n)) = \{f(n) : \exists\ c, n_0 > 0 \text{ such that } f(n) \leq cg(n),\ \forall n \geq n_0\}$.*

Problem 6. *Let $p(n) = \sum_{i=0}^{d} a_i n^i$ be a polynomial of degree d and assume that $a_d > 0$. For $k \geq d$, show that $\lim_{n \to \infty} \frac{\sum_{i=0}^{d} a_i n^i}{n^k} = a_d n^{d-k} \geq a_d > 0$ and conclude that $p(n) \in \Omega(n^k)$.*

Problem 7. *Let $p(n) = \sum_{i=0}^{d} a_i n^i$ be a polynomial of degree d and assume that $a_d > 0$. Show that $p(n) \in \Theta(n^d)$. (Hint: Using limits show that $\lim_{n \to \infty} \frac{p(n)}{n^d} = a_d$ and $0 < a_d < \infty$. Thus, $p(n) \in \Theta(n^d)$. Recall that $f(n) \in \Theta(g(n))$, if $0 < \lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty$.)*

Problem 8. *Show that $6n \log n + \sqrt{n} \log^2 n = \Theta(n \log n)$.*

Problem 9. *Fibonacci numbers are defined recursively as follows:*
$F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for any integer $n > 1$.
Using induction show that $F_n \geq 2^{\frac{n}{2}}$ for any $n \geq 6$.

References

[1] S. DasGupta, C. Papadimitriou, V. Vazirani. *Introduction to Algorithms.* McGraw Hill.

[2] A. Maheshwari. *Notes on Algorithm Design*, Chapter 1, `https://people.scs.carleton.ca/~maheshwa/Notes/DAA/notes.pdf`