Consider the following 2-action automaton:
The automaton has three states \( \{ \phi_i \mid i=0,1,2 \} \).
The automaton has two actions \( \{ \alpha_i \mid i=1,2 \} \).
The F function is defined as follows:
(i) If the automaton is in \( \phi_i \) (i=1,2), on being rewarded it stays in \( \phi_i \) with probability 'b'. It goes to \( \phi_j \) (j \( \neq \) i) with a probability 'a', and goes to \( \phi_0 \) otherwise.
(ii) If the automaton is in \( \phi_0 \), on being rewarded it stays in \( \phi_0 \) with probability 'a' and goes to \( \phi_i \) (i=1,2) with equal probability, otherwise.
(iii) If the automaton is in \( \phi_i \) (i=1,2), on being penalized it goes to \( \phi_j \) (j \( \neq \) i) with probability 'b', stays in \( \phi_i \) with a probability 'a', and goes to \( \phi_0 \) otherwise.
(iv) If the automaton is in \( \phi_0 \), on being penalized it stays in \( \phi_0 \) with probability 'b' and goes to \( \phi_i \) (i=1,2) with equal probability otherwise.

The G function is defined as follows:
If the automaton is in state \( \phi_i \) (i=1,2) it chooses action \( \alpha_i \) with probability 1. If it is in \( \phi_0 \) it chooses both the actions with probability 0.5.

(a) Describe the automaton pictorially and using the \( F^0 \), \( F^1 \) and G matrices.

(b) Describe an equivalent automaton for which the output matrix is deterministic. (Does this machine have to have 6 states??) Note that you must define the new machine, by specifying its states, and its F and G functions. Do this by describing the automaton pictorially and using matrices.

(c) Write down the \( F^- \) matrix of the old automaton with 'a'=0.2 and 'b'=0.7, when it interacts with an environment (0.4, 0.6). If \( \Pi(0) = [0.2, 0.4, 0.4] \), what are \( P(0) \), \( \Pi(1) \) and \( P(1) \) ?

(d) Write down the \( F^- \) matrix of the new automaton under the identical conditions of (c) above. For this machine show that \( P(0) \) and \( P(1) \) are exactly as in the above case.