Consider the two-class pattern recognition problem in which the class conditional distributions are both normally distributed with arbitrary means \(M_1\) and \(M_2\), and covariance matrices \(\Sigma_1\) and \(\Sigma_2\) respectively. Assume that you are working in a 3-D space (for example, as in Assignment II) and that the covariance matrices are not equal.

(a) Generate 200 training points of each distribution before diagonalization and plot them in the \((x_1-x_2)\) and \((x_2-x_3)\) domains.

(b) Using these training points estimate the parameters of each distribution using a maximum likelihood and a Bayesian methodology. In the latter, assume that you know the covariances. Plot the convergence of the parameters with the number of samples in each case.

(c) Using these same training points estimate each univariate distribution using a Parzen Window approach. In this case, work with the features in each dimension separately, and with an appropriate Gaussian kernel. For the output, you must plot the final learned distribution of the features in each dimension, and print out their “sample” mean and variance in each dimension.

(d) Using the estimated distributions, compute the optimal Bayes discriminant function (for the ML, Bayes and Parzen schemes) and plot it in the \((x_1-x_2)\) and \((x_2-x_3)\) domains.

(e) Generate 200 new points for each class for testing purposes, classify them and report the classification accuracy. Do this (i.e., using all 400 points) using a ten-fold cross validation and a leave-one-out method.

(f) Repeat (a)-(d) for the same data after you have diagonalized it.