# Carleton University School of Computer Science Winter 2022 

Consider a two-class problem in which the class conditional distributions are both normally distributed in 3-dimensions with means $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$, where:

$$
\mathrm{M}_{1}=\left[\begin{array}{lll}
4 & 1 & 3
\end{array}\right], \text { and, } \quad \mathrm{M}_{2}=\left[\begin{array}{llll}
-4 & 1 & -3
\end{array}\right] .
$$

The covariance matrices $\Sigma_{1}$ and $\Sigma_{2}$ are :

$$
\Sigma_{1}=\left[\begin{array}{ccc}
a^{2} & \alpha a b & \beta a c \\
\alpha a b & b^{2} & \alpha \mathrm{bc} \\
\beta \mathrm{ac} & \alpha \mathrm{bc} & \mathrm{c}^{2}
\end{array}\right]
$$

and

$$
\Sigma_{2}=\left[\begin{array}{ccc}
\mathrm{c}^{2} & \beta \mathrm{bc} & \alpha \mathrm{ac} \\
\beta \mathrm{bc} & \mathrm{~b}^{2} & \alpha \mathrm{ab} \\
\alpha \mathrm{ac} & \alpha \mathrm{ab} & \mathrm{a}^{2}
\end{array}\right]
$$

(a) Write a program to generate Gaussian random vectors assuming that you only have access to a function which generates Uniform random variables.
(b) Using the strategy taught in class, write a program to simultaneously diagonalize both the distributions. Print out the diagonalizing matrices for a few cases, and in particular, for the case of $a=4, b=3, c=2$ and $\alpha=0.1, \beta=0.2$. Show the intermediate covariance matrices in the process.
(c) Generate 200 points of each distribution for the case of $a=4, b=3, c=2$ and $\alpha=0.1, \beta=0.2$. before diagonalization and plot them in the $\left(\mathrm{x}_{1}-\mathrm{x}_{2}\right)$ and ( $\mathrm{x}_{1}-\mathrm{x}_{3}$ ) domains. These points are 200 3-D vectors, but the projected points in the ( $\mathrm{x}_{1}-\mathrm{x}_{2}$ ) and ( $\mathrm{x}_{1}-\mathrm{x}_{3}$ ) domains must be plotted graphically.
(d) Consider the same 200 generated in (b) above for the case of $a=4, b=3, c=2$ and $\alpha=0.1, \beta=0.2$. after diagonalization and plot them in the $\left(x_{1}-x_{2}\right)$ and ( $x_{1}-x_{3}$ ) domains. Again, remember that these points are 200 3-D vectors, but the points in the $\left(\mathrm{x}_{1}-\mathrm{x}_{2}\right)$ and ( $\mathrm{x}_{1}-\mathrm{x}_{3}$ ) domains must be plotted graphically.

