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August 2002

* published in International Conference on Pattern Recognition (ICPR 200). Volume 2, pp. 312-315. Québec, Québec, Canada. August 2002. NRC 45859.

Some Improvements on Two Autocalibration Algorithms based on the Fundamental Matrix

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Abstract

Autocalibration algorithms based on the fundamental matrix must solve the problem of finding the global minimum of a cost function which has many local minima. We describe a new method of achieving this goal, which uses a stochastic optimization approach taken from the field of evolutionary algorithms. In theory, approaches that use the fundamental matrix for autocalibration are inferior to those based on a projective reconstruction. We argue that in practice if we use this new stochastic optimization approach this is not true. When autocalibrating focal length and aspect ratio both methods achieve comparable results. We demonstrate this experimentally using published image sequences for which the ground truth is known.

1 Introduction

The goal of autocalibration is to find the intrinsic camera parameters directly from an image sequence without resorting to a formal calibration process. The recent interest in autocalibration comes from advances in the field of projective vision which makes it possible to compute various quantities from an uncalibrated image sequence; in particular, the fundamental matrix between image pairs.

In this paper we perform a comparison of two autocalibration algorithms that use fundamental matrices; the first uses Kruppa's equation [1, 2, 3], and the second autocalibrates by optimally converting a fundamental matrix to an essential matrix [4]. We assume that the intrinsic camera parameters are constant over the entire image sequence. In both cases, the problem can be formulated as the minimization of a cost function. The correct camera calibration corresponds to the global minimum of this cost function over the space of possible camera parameters. The claim has been that such minimization approaches to autocalibration are sensitive to the initial starting point of the required gradient descent algorithm [5]. As is shown in the paper when autocalibrating only the focal length, this is not true because we can exhaustively solve the associated 1D optimization problem using standard numerical approaches [6]. We also show that when autocalibrating both focal length and aspect ratio a simple stochastic approach from the field of evolutionary algorithms overcomes this problem [7]. Experiments demonstrate that this stochastic method reliably finds the 2D global minimum.

2 Autocalibration from the Fundamental Matrix

The standard linear camera calibration matrix K has the following entries [2]:

$$K = \begin{pmatrix} fk_u & 0 & u_0 \\ 0 & fk_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

This assumes that the camera skew is $\pi/2$. Here f is the focal length in millimeters, and k_u, k_v the number of pixels per millimeter. The terms fk_u, fk_v can be written as α_u, α_v, the focal length in pixels on each image axis. The ratio α_u/α_v is the aspect ratio. The four free calibration parameters are therefore the focal length α_u, α_v, and the center of projection u_0, v_0. All are in pixel co-ordinates.

The fundamental matrix F is a rank two matrix of size three by three which defines the epipolar geometry between two images [2]. Given a point in one image, the fundamental matrix can be used to compute a line in the other image on which the matching point must lie. The fundamental matrix can be computed from a set of 2D correspondences between two images.

2.1 Equal Singular Values Approach

If we know the camera calibration matrix K, then the essential matrix E is related to the fundamental matrix by $E = K^tFK$. The matrix E is the calibrated version of F; from it we can find the camera positions in Euclidean space. Since F is a rank two matrix, E also has rank two. However, E has the extra condition that the two non-zero singular values must be equal. This fact can be used for autocalibration by finding the K that makes the two singular values of F as close to equal as possible [4]. Given two non zero singular values of E: σ_1 and σ_2 ($\sigma_1 > \sigma_2$), then, in the ideal case $(\sigma_1 - \sigma_2)$ should be zero. Consider the difference $(1 - \sigma_2/\sigma_1)$. If the singular values are equal this quantity is zero. As they become more different, the quantity approaches one. Given a fundamental matrix, autocalibration proceeds by finding the calibration matrix K which minimizes $(1 - \sigma_2/\sigma_1)$.

Assume we are given a sequence of n images, along with their fundamental matrices. Then F_i, the fundamental matrix relating images i and $i+1$, has non zero singular values σ_{i1} and σ_{i2}. To autocalibrate from these n images using the equal singular values method we must find the K which minimizes $\sum_{i=1}^{n-1} w_i(1 - \sigma_{i2}/\sigma_{i1})$. Here w_i is a weight factor, which defines the confidence in a given fundamental matrix. The weight w_i is set in proportion to the number of matching 2D feature points that support the fundamental matrix F_i. The larger this number, the more confidence we have in that fundamental matrix.

2.2 Kruppa's Equation Approach

Another way to perform autocalibration from a set of fundamental matrices is to use Kruppa's equation [2]. To understand this method we must first define the absolute conic. In Euclidean space the absolute conic lies on the plane at infinity. It can be shown that a 2D image point belongs to the projected image of the absolute conic if and only if it lies on the conic represented by matrix $K^{-T}K^{-1}$. Similarly, KK^T is the dual of the image of the absolute conic, and is labeled as C. If we can find C, then we can directly compute the camera parameters K by Cholesky factorization.

Kruppa's equation relates the fundamental matrix to the terms of the absolute dual conic. Recently a new derivation of Kruppa's equation based on the singular value decomposition (SVD) was described [2]. Consider the SVD of a fundamental matrix $F = UDV^T$. Here D is a diagonal matrix of the singular values $(r, s, 0)$. The column vectors of U are u_1, u_2, u_3, and the column vectors of V are v_1, v_2, v_3. Then the new form of Kruppa's equation is $\frac{v_2^T C v_2}{r^2 u_1^T C u_1} = \frac{-v_2^T C v_1}{rs u_1^T C u_2} = \frac{v_1^T C v_1}{s^2 u_2^T C u_2}$. Given the fundamental matrix F we can compute its SVD, and set up these three ratios. The unknown values are the elements of C. To autocalibrate we must find the C which makes these three ratios as close to being equal as possible. Let ra_1 be defined as $\frac{v_2^T C v_2}{r^2 u_1^T C u_1} - \frac{-v_2^T C v_1}{rs u_1^T C u_2}$ with ra_2, ra_3 defined in the same fashion as the other two possible differences of these ratios. Then autocalibration can be achieved by finding the C which minimizes $ra_1^2 + ra_2^2 + ra_3^2$. Assume as before that we are given n images along with their fundamental matrices. The Kruppa ratios for images i and $i-1$ are labeled as $ra_{i1}, ra_{i2}, ra_{i3}$. Then to autocalibrate over n images we must find the C which minimizes $\sum_{i=1}^{n-1} w_i(ra_{i1}^2 + ra_{i2}^2 + ra_{i3}^2)$. Here again the weights w_i represent our confidence in the given fundamental matrix as defined in the end of the previous section.

2.3 Numerical Optimization

The two autocalibration approaches we have described require the minimization of a cost function of the calibration parameters. In theory a gradient descent algorithm can find the solution. The problem is that there are often many local minima in the cost function, so the solution that is found depends on the starting point of the gradient descent algorithm. However, we note that the calibration parameters can all be bounded; i.e. the center of projection is rarely more than one fifth of the image size from the image center. Thus we are need to find the global minimum of a set of real-valued, bounded optimization parameters. This problem has been dealt with in the field of evolutionary algorithms.

We use an approach called dynamic hill climbing (DHC) which is very successful in solving such real valued optimization problems [7] . It performs repeated gradient descent in the search space, but restarts the descent as far as possible from previous solutions. A single gradient descent of the cost function uses the Powell optimization algorithm [6]. The pseudo-code for the DHC optimization algorithm is as follow:

```
For n times
  For each free optimization parameter
  in the calibration matrix
    Find the largest contiguous region not
      containing a previous local optimum
      of the gradient descent.
    Choose a random point in this region
      as the new starting point.
  Endfor
  Run the gradient descent algorithm from
    this new starting point.
  Save the best cost function value.
Endfor
Return the best calibration parameters.
```

Name of Sequence	True focal	Eigen focal	Kruppa focal
castle	1100	1156.50	1197.70
valbone	682	605.50	685.71
nekt	700	798.58	872.44
etlueshiba	837	857.25	1233.85

Table 1: Autocalibration of focal length (in pixels) for published sequences.

Name of Sequence	True focal	Eigen focal	Kruppa focal
curve	6.97	4.71	7.49
cylinder	28.0	26.35	31.70
plant	24.20	22.55	24.39
statue	5.11	3.67	5.29

Table 2: Autocalibration of focal length (in mm.) for photogrammetric sequences.

3 Experimental Results

For many autocalibration algorithms, performance evaluation consists of a simple visual inspection of a 3D reconstruction based on the computed camera calibration. This is not adequate because the quality of the final reconstruction is visually acceptable for a wide variety of calibration parameters [8]. Instead we perform tests on image sequences for which the ground truth camera calibration is known. We experiment with two different types of image sequences; one where we are given the correspondences a-priori, and one where we must compute the correspondences automatically from the images. In both cases, the fundamental matrix is calculated from these correspondences. The software used in our experiments is part of the Projective Vision Toolkit and is available on our web page [9]. It can compute the fundamental matrix from a given set of correspondences, and can automatically find a reliable set of correspondences between image pairs.

The first experiment demonstrates the autocalibration of only the focal length. Table 1 shows results for a number of sequences which have been processed in previously published autocalibration papers [1, 3, 10, 11]. In particular, the castle sequence is used as a test case for the autocalibration approach which requires a projective reconstruction [10]. In these experiments the 2D features used to compute the fundamental matrices were found automatically from the images using our software.

In the next experiment, 2D features were selected by hand as part of a model building process using a well known photogrammetric package [12]. The assumption is that they are correct correspondences. We know all the intrinsic parameters of the camera a-priori and assume they are constant. The exception is the focal length, which we autocalibrate. Table 2 shows the autocalibrated focal length in millimeters versus the true focal length. Another measure of the quality of the autocalibration are the reprojection errors. These are the pixel differences between the 2D projections of the reconstructed 3D feature points and their original corresponding 2D feature points. This can be computed because we have created a 3D reconstruction of these 2D features. For each focal length we calculate the reprojection errors for all the points, sort these errors, and save the median value. This median is a good indicator of the quality of the 3D reconstruction created using a given focal length. We repeat this calculation for all the focal lengths in Table 2. The median value of the reprojection error of all four sequences using the correct focal length is 1.7 pixels, with the focal length from the singular value method is 3.85 pixels and from Kruppa's method is 1.75 pixels. We see that using the autocalibrated focal lengths to create a 3D reconstruction increases the reprojection errors only slightly versus the true focal lengths.

In the final experiment, we autocalibrate both aspect ratio and focal length using as input the same images sequences listed in Table 2. The results, as shown in Table 3, demonstrate that the errors when autocalibrating two camera parameters are sometimes higher than when autocalibrating just one parameter. One possible explanation is that the gradient descent algorithm is stuck in a local minima. To check this hypothesis the results shown in Table 3 were computed by averaging over one hundred separate runs of our optimization algorithm. The variance for the computed aspect ratio is 0.00391 and for the computed focal length is 0.1735. The stochastic process starts each gradient descent in a different part of the search space. This implies that if it were converging prematurely then this local minimum would change. Therefore the low variance indicates that the true global minimum is being found.

4 Conclusions

In theory, autocalibration methods that use fundamental matrices should not perform as well as those

Name of Sequence	True aspect	Eigen aspect	True focal	Eigen focal
curve	1.0	1.08	6.97	3.46
cylinder	1.0	0.98	28.0	26.72
plant	1.0	0.98	24.2	22.96
dam	0.81	0.972	30.75	38.52

Name of Sequence	True aspect	Kruppa aspect	True focal	Kruppa focal
curve	1.0	0.997	6.97	7.56
cylinder	1.0	1.03	28.0	32.91
plant	1.0	0.92	24.2	26.33
dam	0.81	0.997	30.75	38.43

Table 3: Autocalibration of focal length (in mm.) and aspect ratio for photogrammetric sequences.

that use the camera projection matrices of a projective reconstruction [13]. However, we show that for non-degenerate motions with fixed camera parameters this is not the case. Both the equal singular value approach and Kruppa's equations approach perform as well as all the methods published in the literature when calibrating only the focal length, or the focal length and aspect ratio. One possible explanation for the good results using Kruppa's approach is that using the SVD based cost function [2] is superior to the cost function which requires the computation of the camera epipoles [13]. The equal singular values approach is very simple and works just as well as Kruppa's method. It perform better than Kruppa's method in situations where we are close to a degenerate motion, such as pure translation. Using two different autocalibration methods has the advantage of increasing confidence in the results when both answers are similar.

Computationally the fundamental matrix based approaches are very efficient since a single evaluation of the cost function does not take long to compute. The total time taken for autocalibration is in the order of seconds for all the experiments. Some approaches to autocalibration require the solution to a set of polynomial equations [10], but this is not computationally feasible for long image sequences. With our optimization based approach we can efficiently process long image sequences, which is an advantage. One argument against the optimization based methods has been that they are sensitive to the starting point of the gradient descent algorithm [5, 14]. We have shown that when using our stochastic optimization approach this is not the case. The error in the autocalibration of the focal length is usually in the range of 5% to 10%. This is adequate for applications such as visualization or model building.

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